Utilitarian Preference for Equality: The Case Against Max-Min

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Abstract

This paper studies an optimal linear tax or basic income regime with an extensive margin in the supply of labor, but without a weighting function embodying social welfare. Some redistribution maximizes aggregate utility, but a lesser degree than what Rawls’ max-min criterion would require. The presence of heterogeneity breaks the connection between allocational efficiency and aggregate utility maximization. Rawls’ concept of the Original Position is useful, but the max-min criterion is questionable.

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1 Introduction

Though the thought of sacrificing the well-being of the least fortunate in a group is repugnant to most, the response to The Lottery [Jackson, 1948] being a prominent example, developing a rigorous method for determining a desirable degree of redistribution is not trivial. The introduction of heterogeneity into macroeconomic models raises important questions about the proper welfare criterion and the definition of efficiency. The goal of the present paper is to develop a simple model of redistributive taxation and examine the associated political economy issues.

In an environment where wages are heterogeneous due to differences in worker productivity, the government collects income taxes via a flat marginal rate and redistributes a lump sum transfer to all agents, i.e. a negative income tax or basic income plan. We allow for the possibility that the income tax may drive some low productivity workers out of the labor force and restrict attention to utilitarian formulations of welfare based on aggregate utility, eschewing social welfare transformations that represent a social preference for equality. The primary contribution of the paper is to compare the optimal redistribution regime under utilitarian welfare criteria and the Rawlsian max-min criterion.

The focus here is on utilitarian welfare criteria that sum the utilities across agents. Under diminishing marginal utility, there is an inherent preference for equality, a point often overlooked. It is common to define welfare using a concave transformation across utilities, where the concavity represents a societal preference for equality above that given by the utilitarian criterion. However, whether society-wide agreement about the preference for equality is possible or meaningful is questionable and such transformations are necessarily arbitrary.

Rawls [1971] advocates for the max-min criterion, where the outcome of the least fortunate member of society is maximized. With the social welfare transformation, the max-min criterion is the limiting case with respect to the concavity of the function. Without such transformations, the optimal regime under the max-min criterion is treated separately and is simply the tax rate at the peak of the Laffer curve.

Proposition 7 shows that degree of redistribution satisfying the max-min criterion is necessarily greater than the one that maximizes aggregate utility. The model formalizes the objection of some philosophers to the max-min criterion that rational agents behind the veil might be willing to risk a bad outcome. Given minimal restrictions on household preferences, the max-min criterion is not necessary to justify redistribution and yields sub-optimal aggregate utility. Demonstrating this result for general household preferences without reference to social welfare transformations is the primary contribution of the paper.

The optimal outcome under a utilitarian (Benthamite) welfare criterion has a number of appealing properties. Maximizing aggregate utility can be interpreted as the decision of a representative agent behind the
"veil of ignorance," the Original Position in Rawls [1971]. For decreasing marginal utility, redistribution is desirable since the gain to low wage workers outweighs the loss for high wage workers. The basic income regime is not subject to some common criticisms of redistribution. Under the linear (flat rate) tax, workers do not have incentive to take lower wage jobs to avoid taxes. Also, the lump sum transfer is given to all so there is no distortion of labor supply decisions.

The presence of heterogeneity in wages and productivity breaks the equivalence between welfare maximization and the usual allocational efficiency condition under a representative agent. In the present model, allocational efficiency is achieved when the tax rate is zero and there is no redistribution. As noted, maximum aggregate utility is achieved for some degree of redistribution for a reasonable specification of the model.

These results agree with those found in the standard optimal linear tax model, summarized in Piketty and Saez (2013), but the model here relies only on standard household utility preferences, not on diminishing marginal welfare across utilities on the part of the social planner. The standard approach includes an arbitrary weighting function on the aggregate utilities as opposed to a strictly utilitarian welfare function. There are interesting qualitative statements that arise from such a model, but the justification for the inclusion such a social welfare weighting function is arbitrary, and some results lose relevance.

Individually, the elements of the present framework are not novel, but the combination and the focus on the interpretation of the welfare criteria is. Optimal flat rate or linear tax regimes are studied in Stiglitz [1985] and Dixit and Sandmo [1977]. The labor supply margin is included in the models in Mirlees [1971] and Diamond [1980]. Atkinson [1973] analyzes optimal taxation under the max-min criterion. Manski [2012] also considers simple tax regimes in a model with government production that affects consumption and productivity. He discusses informally some of the issues around efficiency in the present work.

The paper is organized as follows. Section 2 describes the elements of the model and some of the desirable properties of the basic income regime. Section 3 discusses the formalization of the welfare criteria. Section 4 reports the primary formal results of the paper related to optimal tax rate. Sections 5 and 6 introduce a particular calibration of the model to show the quantitative relevance of the results and their relation to the literature on income distribution. Section 7 concludes.

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2 Also, the model presented here has an explicit production sector, unlike some of the models described in Piketty and Saez [2013].
2 The Model

The key elements of the model are heterogeneous productivity and the extensive labor supply margin. A unit mass of households indexed by $i$ have heterogeneous productivity $a_i$ with single-peaked density $a_i \sim f(a_i) \in (0, \infty)$ and associated distribution $F(a_i)$. Labor markets are competitive so real wages $w_i$ are determined by productivity such that $w_i = a_i$. Labor is the only input in the model. Each household supplies quantity of labor $h_i$ which produces output $a_i h_i$, the production function for a single input with constant returns to scale. Aggregate production $Y$ is thereby

$$Y = \int_0^\infty a_i h(a_i) dF(a_i).$$  \hfill (1)

Households receive wage income, pay taxes at a marginal rate of $\tau$ and receive a lump sum transfer $T = \tau Y$. The household budget constraint is

$$c(a_i) = a_i h(a_i) (1 - \tau) + \tau Y$$  \hfill (2)

where $c(a_i)$ is household consumption. Both consumption and hours worked are functions of an individual’s productivity $a_i$. Since there is a unit mass of agents, the values $c(a_i), h(a_i), Y$ and $T$ can be interpreted as per capita values. For positive tax rates $\tau > 0$, there is redistribution of income from high wage (productivity) to low wage (productivity) households so that all households have a minimum income $\tau Y$.

Households maximize utility $U^i(c(a_i), h(a_i))$ that is a function of consumption and leisure, which is the time left over from working hours. We make minimal assumptions about the form of the utility function: $U^i_1 > 0, U^i_{11} \leq 0, U^i_2 < 0, U^i_{22} > 0$.

Changes in consumer behavior have an infinitesimal effect on the government transfer. Such a lump sum transfer does not distort individual labor supply decisions, effectively implementing a basic income plan or negative income tax, advocated by Milton Friedman\(^3\) among others.

The pair $(c^*(a_i), h^*(a_i))$ satisfies the optimality condition given by maximizing $U_i(c(a_i), h_i(a_i))$ subject to the household budget.

$$U^i_1(c^*(a_i), h^*(a_i)) a_i (1 - \tau) = U^i_2(c^*(a_i), h^*(a_i))$$  \hfill (3)

The standard allocational efficiency condition is a special case of the household optimality condition above. The representative agent case arise for the degenerate density $f(\bar{a}) = 1$ for any $\bar{a} > 0$. Maximizing

\(^3\)Friedman [1962] presents the negative income tax as a fixed deduction from wages that are taxed, which is equivalent to the regime here for any given tax rate. See also http://www.basicincome.org/bien/.
utility under the production constraint $Y = \bar{m}h$, motivates the following.

**Definition 1** The *allocational efficiency* condition is satisfied where household utility is maximized under the constraint (2) for a representative level of productivity $\bar{a}$.

Allocational efficiency is achieved as a special case of the household optimization condition (3) for $a_i = \bar{a}$ and $\tau = 0$, corresponding to the standard efficiency concept that the marginal utility of a good equals the marginal cost of production. However, for this condition to hold under heterogeneous productivity for all $a_i \in (0, \infty)$ requires further restriction. The marginal utility of consumption is always strictly positive $U_i'(c) > 0$, and workers would not work at a wage of zero $h^*(0) = 0$. Hence, for the above condition to apply in the neighborhood of $a_i = 0$, it must also be the case that the marginal disutility of labor is zero at that point, $U_i^2(c(0),0) = 0$. This discussion is summarized in the following remark.

**Remark 2** In the model given by equations (1), (2) and (3), the allocational efficiency condition (Definition 1) is achieved for all workers $i$ at a marginal tax rate of zero $\tau = 0$, if the marginal disutility of labor at zero hours worked is zero, $U_i^2(c(0),0) = 0$.

In the presence of heterogeneity, allocational efficiency is achieved in the absence of redistribution when labor is supplied by all households, but the correspondence with optimal welfare is not guaranteed. Furthermore, the assumption about the marginal disutility of labor in Remark 2 is questionable. If that assumption does not hold, working negative hours makes little sense so let the labor supply function be

$$h(a_i) = \max(0,h^*(a_i)),$$

which, along with the budget constraint (2), determines the consumption function $c(a_i)$. The bound on hours implies that there is a productivity threshold below which workers drop out of the labor market and their only income is the transfer from the government.

**Definition 3** The productivity threshold $\bar{a}$ is such that $h^*(a_i) \leq 0$ for all $a_i \leq \bar{a}$.

There are a number of additional reasons to focus on a tax and redistribution system of the present form. The transfer applies to all agents so there is no need for means-testing, which is costly and raises the potential for some to misrepresent their level of productivity. It also avoids problems of envy found in the optimal tax literature and most prominently in the model with two labor types in Stiglitz [1982].

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4See Stiglitz (1987) and Piketty and Saez (2013) for a full discussion.
Proposition 4  Household utility $U^i(c(a_i), h(a_i))$ is non-decreasing in $a_i$ for any tax rate $\tau$.

Proof. See Appendix A. ■

The tax-transfer regime under consideration here does not lead to envy among high productivity (wage) workers, and the lump sum transfer does not distort labor markets as noted above. Furthermore, it does not require much sophistication on the part of the government to implement. Hence, this tax-transfer specification is one of the least objectionable redistribution schemes.

Not to say that taxes do not have adverse effects. On the upward sloping portion of the labor supply curve the threshold $\bar{a}$ is increasing in $\tau$, so as the tax rate increases workers drop out of the labor market.

3 Welfare

Welfare is defined without an explicit preference for equality. Aggregate utility is one obvious choice of welfare criterion to judge the effect of redistribution, following most of the optimal tax literature including Mirlees [1971], Manski [2012] and Werning [2007]. The Rawlsian approach to welfare is treated separately.

Aggregate utility $U^e$ is specified by aggregating household utility over the distribution of productivity $f(a_i)$.

$$U^e = \int_0^\infty U^i(c(a_i), h(a_i)) dF(a_i)$$

$$= \int_0^{\bar{a}} U^i(\tau Y, 0) f(a_i) + \int_{\bar{a}}^\infty U^i(c(a_i), h(a_i)) dF(a_i)$$

(5)

The second line expresses separately the utility of those with productivity below $\bar{a}$ who do not work and who consume only the transfer $\tau Y$.

Unlike many models in this literature, we do not include an arbitrary weighting function representing preferences the government or social planner. Such models introduce a concave function $G(\cdot)$ and define welfare as $\int_0^\infty G(U^i) dF(a_i)$. Concavity of the weighting function $G(\cdot)$ indicates diminishing marginal social welfare across the utilities of the population with the limiting case $G'(0) = \infty$ representing the Rawlsian preference to maximize the utility of the least productive agent. While such an approach is mathematically elegant, the weighting function is arbitrary, and the results that follow could be dismissed.

One could also consider aggregate utility $U^e$ to be the utility of a representative agent behind the "veil of ignorance," meaning he does not know his own draw from the distribution $F(a_i)$. Rawls uses this concept in his development of principles of justice from the "Original Position", which would be agreed upon by citizens behind the veil. Werning [2007] considers aggregate utility to be a representation of insurance for
households against poor outcomes. In the present context, one could regard as fair any system that yields aggregate utility above that achieved in a state of nature, though formalization of this idea is beyond the scope of the present work.

Rawls’ concept is a "thick veil", where agents have no knowledge of the structure of the economy, which leads him to argue in favor of the stronger max-min criterion, which requires that the utility of the least advantaged agent is maximized. In the present context, the max-min principle is satisfied by the tax rate at the peak of the Laffer curve\(^5\), which relates the transfers (government revenue) \(T(\tau)\) and the tax rate \(\tau\).

The transfer is also the only income of the least productive worker at \(a = 0\), so the tax rate that maximizes \(T(\tau)\) satisfies the max-min principle.

The present work considers the role of redistribution on welfare for both the max-min criterion and aggregate utility. The following derivative shows the effect of redistributive taxation on aggregate utility.

The calculation involves multiple applications of Leibniz Rule, since the threshold \(\tilde{a}\) depends on the tax rate \(\tau\). Also note that \(F'(a) = f(a)\).

\[
\frac{dU^c}{d\tau} = U_1(\tau Y, 0) \left( Y + \tau \frac{dY}{d\tau} \right) F(\tilde{a}) + U_1(\tau Y, 0) \frac{d\tilde{a}}{d\tau} \int_0^\infty \left[ U_1^1(c(a_i), h(a_i)) \frac{dc_i}{d\tau} + U_2^1(c(a_i), h(a_i)) \frac{dh_i}{d\tau} \right] dF(a_i) - U_1(c(\tilde{a}), h(\tilde{a})) f(a) \frac{d\tilde{a}}{d\tau}
\]

The effects of an increase in the marginal tax rate \(\tau\) expressed above are intuitive. The \(\frac{d\tilde{a}}{d\tau}\) show the effect of workers exiting the workforce, the first term shows the increased utility of those receiving only the transfer, and the integral term shows the change in utility for those in the workforce.

The definition of \(\tilde{a}\) (Definition 3) implies that for \(a_i < \tilde{a}\), it is the case that \((h(a_i)) = 0\) and \(c(a_i) = \tau Y\). Hence, the \(\frac{d\tilde{a}}{d\tau}\) terms cancel and the above expression simplifies.

\[
\frac{dU^c}{d\tau} = U_1(\tau Y, 0) \left( Y + \tau \frac{dY}{d\tau} \right) F(\tilde{a}) + \int_0^\infty \left[ U_1^1(c(a_i), h(a_i)) \frac{dc_i}{d\tau} + U_2^1(c(a_i), h(a_i)) \frac{dh_i}{d\tau} \right] dF(a_i)
\]

Using the household optimality condition (3) and differentiating the household budget constraint (2) with respect to \(\tau\) and substituting yields alternative forms for \(\frac{dU^c}{d\tau}\).

\[
\frac{dU^c}{d\tau} = U_1(\tau Y, 0) \left( Y + \tau \frac{dY}{d\tau} \right) F(\tilde{a}) + \int_0^\infty \left[ U_1^1(c(a_i), h(a_i)) \left( Y - a_i h(a_i) + \tau \frac{dY}{d\tau} \right) \right] dF(a_i)
\]

\(^5\)Piketty and Saez [2013] note that this concept can be traced to Jules Depuit.
4 Redistribution

Some degree of redistribution maximizes aggregate utility given minimal assumptions. However, the aggregate utility maximizing tax rate is less than the rate that satisfies the Rawlsian welfare criterion.

Another form of the derivative \( \frac{dU^e}{d\tau} \) is obtained by combining the two terms in the relation (6), using the fact that the first term does not depend on \( a_i \) for \( a_i \in (0, \bar{a}) \).

\[
\frac{dU^e}{d\tau} = \int_0^{\infty} U_1^i (c(a_i), h(a_i)) \left[ Y - a_i h(a_i) + \tau \frac{dY}{d\tau} \right] dF(a_i) \tag{7}
\]

A trivial but important special case is that of constant marginal cost \( U_1^i (\cdot) = \tau \), where the above reduces to \( \frac{dU^e}{d\tau} = \tau \frac{dY}{d\tau} \). Output is decreasing in the tax rate, so \( \frac{dU^e}{d\tau} \leq 0 \). Further, at the zero tax rate it is the case that \( \frac{dU^e}{d\tau} |_{\tau=0} = 0 \) under constant marginal utility so the introduction of redistribution does not improve welfare. Hence, any argument in favor of redistribution depends on decreasing marginal utility. Of course, assuming constant marginal utility across all income levels is highly questionable.

For realistic specifications of the utility function, some degree of redistribution is desirable. Using the equilibrium condition\(^6\) \( Y = \int_0^{\infty} c(a_i) f(a_i) da_i \) gives the following.

**Proposition 5** Given that marginal utility is decreasing in \( a_i \in (0, \infty) \), \( U_1^i (c(a_i), h(a_i)) < 0 \) and consumption is increasing in productivity, \( c'(a_i) > 0 \), some degree of redistribution increases aggregate utility such that \( \frac{dU^e}{d\tau} |_{\tau=0} > 0 \).

**Proof.** See Appendix A. \( \blacksquare \)

For redistributive taxation to have any benefit, the marginal utilities and mass of agents whose productivity (and wage) places them below mean income must be greater than those above. The above result is neither surprising nor novel, though the precise framework and construction of the proof is.

Even with the restrictions of a fixed marginal tax and lump sum transfer for all, redistribution is beneficial, as in Mirlees [1971]. The assumption that consumption is increasing in productivity is equivalent to the assumption that it is not a Giffen good. It might be violated for an extremely backward bending labor supply curve at higher levels of productivity, but the case for redistribution would be even stronger here. Workers on the backward bending portion of the labor supply curve would supply more labor at a higher tax rate.

**Corollary 6** For the assumptions in Proposition 5, the tax rate that maximizes aggregate utility does not satisfy the allocational efficiency condition in Remark 2.

\(^6\)This condition can be recovered by integrating the household budget constraint as well.
**Proof.** The efficiency condition is met for $\tau = 0$, and aggregate utility is maximized at $\tau > 0$ as shown in Proposition 5. ■

This result clouds all arguments that rely on the idea that allocational efficiency is desirable. While some concept of efficiency properly defined may be useful, the role of heterogeneity is crucial.

Now that it has been established that a positive tax rate maximizes aggregate utility, the primary result that the Rawls’ max-min criterion calls for an even greater degree of redistribution can be shown.

**Proposition 7** Given that the unique tax rate that maximizes aggregate utility $U^e$ is $\tau^*$, and that the rate that satisfies the max-min criterion is $\overline{\tau}$, then

$$\tau^* < \overline{\tau}.$$ 

**Proof.** Hours $h(a_i)$ increase with wages at a level of wages sufficiently low. Therefore, for a sufficiently high tax rate $\tau$, $h(a_i)$ decreases with $\tau$. Further, the household optimality condition (3) and labor supply specification (4) implies that if $\tau = 1$, $h(a_i) = 0$ for all $a_i$. Hence, for $\tau$ sufficiently large, output $Y$ is decreasing with $\tau$, and the Laffer curve has its usual shape where $T(\tau)$ has a single maximum and $T(0) = T(1) = 0$.

The proof proceeds by contradiction. Assume $\tau^* > \overline{\tau}$. Since, both functions of the tax rate $U^e(\tau)$ and $T(\tau)$ have a unique maximum on $\tau \in [0,1]$, it follows that $\frac{dU^e}{d\tau}|_{\tau=\overline{\tau}} > 0$ and $\frac{dT}{d\tau}|_{\tau=\overline{\tau}} = 0$. The latter equality implies $(Y + \tau \frac{dY}{d\tau})|_{\tau=\overline{\tau}} = 0$. Combining these relations with the expression (7) yields

$$U_1^i(c(a_i), h(a_i)) \left[ Y - a_i h(a_i) + \tau \frac{dY}{d\tau} \right] dF(a_i)|_{\tau=\overline{\tau}} > 0,$$

but this can be written

$$-U_1^i(c(a_i), h(a_i)) a_i h(a_i) dF(a_i)|_{\tau=\overline{\tau}} + U_1^i(c(a_i), h(a_i)) \left[ Y + \tau \frac{dY}{d\tau} \right] dF(a_i)|_{\tau=\overline{\tau}} > 0.$$

Since $(Y + \tau \frac{dY}{d\tau})|_{\tau=\overline{\tau}} = 0$, $U_1^i(\cdot) > 0$ and all the other terms in the first integral are positive, the above relation cannot be true. Therefore, it must be the case that $\tau^* < \overline{\tau}$ as required. ■

In an economic environment with minimal restrictions, the aggregate utility maximizing degree of redistribution is less than that determined by the max-min principle. Alternatively, satisfying the max-min principle would require a reduction in aggregate utility as compared to the optimum. The proof makes the reason clear. Considering the expression for aggregate utility (5), the max-min principle requires maximizing...
the first integral over $[0, \bar{a}]$ while ignoring the second integral over $[\bar{a}, \infty]$, meaning the utility of the least productive is maximized without regard to the effect on the rest of the population.

The results that the Rawlsian marginal tax is revenue maximizing and higher than the welfare maximizing tax rate agree with results from the standard optimal linear tax model in Picketty and Saez (2013) that includes social welfare weighting function. Proposition 7 also parallels results in Werning [2007], which derives conditions for Pareto efficient income tax regimes. The resulting income distribution for such a regime must stochastically dominate the distribution that satisfies the Rawlsian (max-min) criterion for social welfare.

Proposition 7 formalizes the objection to the max-min criterion that, even for someone deciding behind the veil of ignorance, a person might be willing to risk a poor outcome if the probability is sufficiently small. Rawls' [1971] argues that such a risk would not be rational but is not wholly convincing, see Nagel [1973] and Scanlon [1973]. The view embodied in the present model is that the Original Position is a useful concept for evaluating policy and societal norms, but relying on the max-min criterion is suspect.

A defender of the max-min criterion would argue that the veil in the above model is not sufficiently thick in that agents have knowledge about their preferences and the structure of production in the economy. Though the assumptions about the utility function are quite general, it is true that the productive sector is stylized. However, there is no obvious extension that would overturn the result in Proposition 7, though this is an area for future research. More importantly, if the goal is to provide a rigorous framework for fairness, redistribution can be justified on utilitarian grounds with a thin veil, as in the model presented here, and there is no need to discuss the thickness of the veil, which is inherently vague and unlikely to be convincing for many.

5 A Specific Utility Function

To assess the quantitative importance of Proposition 7, we adopt a standard specification of the utility function. Given this specification, the condition for the existence of beneficial redistribution can be expressed in terms of the moments of the distribution of productivity. The condition is met by any reasonable distribution.

Let household utility take the following log-log form.

$$U_i(c_i, h_i) = \log c_i + \psi \log (1 - h_i)$$

(8)

This form is a benchmark case for macroeconomic analysis, Cooley and Prescott [1995] for example. The
parameter $\psi > 0$ represents the relative importance of consumption utility and labor disutility and is calibrated commonly to a value greater than one.

Given the above, the household optimality condition (3) and budget constraint (2) can be used to solve out $c_i$ to find the following labor supply relation.

$$h_i(a_i) = \max \left[ 0, \frac{1}{1+\psi} - \frac{1}{a_i} \left( \frac{\psi}{1+\psi} \right) \left( \frac{\tau}{1-\tau} \right) Y \right]$$

In turn, this relation with the budget constraint (2) determines the consumption function recalling that the threshold $\bar{a}$ is such that $h(a) = 0$ for $a < \bar{a}$.

$$c_i(a_i) = \left( \frac{1-\tau}{1+\psi} \right) a_i + \left( \frac{\tau}{1+\psi} \right) Y \quad \text{for } a_i \geq \bar{a}$$

$$= \tau Y \quad \text{for } a_i < \bar{a}$$

Consumption is increasing in productivity, so the condition in Proposition 5 is satisfied. Note that in the absence of redistribution ($\tau = 0$), the second term under $a_i > \bar{a}$ disappears and labor supply does not depend on wages/productivity. Income and substitution effects of a wage change cancel, the standard result for the log-log specification\(^8\). Hence an increase in taxes negatively impacts the labor supply of all workers in the labor market, and output falls.

To solve for output $Y$ and the threshold $\bar{a}$, multiply both sides of the labor supply relation (9) by $a_i$. Integrating both sides of the equation over the distribution $f(a_i)$ for the interval $[\bar{a}, \infty]$ determines one condition for $\bar{a}$ and $Y$ and the Definition 3 gives another. See the Appendix A for details.

$$Y = \left( \frac{1-\tau}{1-\tau + \psi} \right) \int_{\bar{a}}^{\infty} a_i dF(a_i)$$

$$\bar{a} = \left( \frac{\psi\tau}{1-\tau} \right) Y$$

Together, these relations determine $\bar{a}$ and $Y$ as functions of the parameters $\tau$ and $\psi$. For any level of taxation $\tau > 0$, some workers drop out of the labor supply, as $\bar{a} > 0$. From these relations it is possible to show the effect of taxation.

**Lemma 8** For the log-log utility specification (8), output $Y$ is decreasing in the tax rate $\tau$, and the threshold for worker participation $\bar{a}$ is increasing in $\tau$ for any $\tau \in [0,1)$.

**Proof.** See Appendix A. ■

\(^8\)The Frisch elasticiy is decreasing in labor.
In spite of the negative effects of taxation, some degree of redistribution maximizes aggregate welfare, as guaranteed by Prop 4. For the model with log-log utility, this result can be expressed as a simple restriction on the distribution of productivity.

**Proposition 9** Under the log-log utility specification (8), where the distribution of productivity (wages) $F(a_i)$ has $r$th moment $\mu_r$, some degree of redistribution increases aggregate utility, formally, $\frac{dU^e}{d\tau}|_{\tau=0} > 0$, if $\mu_1\mu_{-1} > 1$.

**Proof.** See Appendix A. ■

Even though the introduction of redistributive taxation means some workers drop out of the work force, the remaining workers work less and aggregate output falls, the overall welfare effect is positive. The restriction on the distribution $F(a_i)$ is met for any associated, single peaked density $f(a_i)^\sim(0, \infty)$, see Appendix B. The condition in Proposition 9 can be expressed in terms of Generalized Entropy and the Atkinson Index, which are standard measures of inequality, see Appendix B.

### 6 Sample Calibration

Calibrating the model with standard parameter values allows us to further examine the welfare implications of the model. Aggregate utility can be computed across the full range of tax rates to determine the optimal rate of taxation and compare it with the rate given by the Rawlsian criterion.

The first calibration task involves the specification of the density of productivity $f(a_i)$. Following Pinkovsky and Sala-i-Martin [2009], let the function $f(a_i)$ take the form of the gamma distribution in Corollary 10 in the Appendix B. Using income data for the U.S. in 2010, estimation of the parameters by calculating the two moments\(^9\) of the data give the maximum likelihood estimate of the parameters. The choice of the utility function (8) parameter $\psi$ is standard, as in Cooley and Prescott [1995].

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\theta$</th>
<th>$\psi$</th>
</tr>
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<tbody>
<tr>
<td>1.47</td>
<td>25.2</td>
<td>2.0</td>
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Given this information, equations (8)-(12) determine aggregate utility $U^e(\tau)$, equation (5), and the transfer $T(\tau) = \tau Y(\tau)$ as functions of the tax rate $\tau$. Figure 1 shows the transfer function and the (monotone) exponential transformation of the aggregate utility function. The transformation is made to ease comparison of the two. As shown by Proposition 7, the tax rate satisfying the max-min criterion at\(^9\) The two moments are the mean of the incomes and the mean of the logs of the incomes. The income data is measured in thousands of dollars.
the peak of the Laffer Curve $\tau = 0.5195$ is higher than the rate that maximizes aggregate utility $\tau = 0.3546$. Furthermore, the result that redistributive taxation is welfare improving is not an artifact of the behavior of the model near the tax rate of zero.

The graph shows the aggregate utility function $\exp(U^e(\tau))$ with maximum at $\tau = 0.3546$ and the transfer (Laffer Curve) $T(\tau)$ with maximum at $\tau = 0.51950$.

The difference in the two tax rates is large, so conclusions from Proposition 7 are quantitatively significant\(^{10}\).

### 7 Conclusion

There is no welfare criterion that is above criticism. It is more important to select a criterion that is reasonable across a wide range of preferences, than one that fits closely with a particular viewpoint. The utilitarian criterion (aggregate utility) does not capture all aspects of well-being, but has the advantage of

\(^{10}\)These results are consistent with those of Tuomala (1984) in a related model.
simplicity and does not impose an extra preference for equality that has obvious implications for questions of redistribution. In identifying an appropriate welfare criterion, Rawls’ concept of the Original Position is useful in that it provides a foundation for the meaning of aggregate utility. However, his assertion that the max-min criterion is rational is suspect. Under minimal assumptions about household preferences, the marginal tax rate that satisfies the max-min criterion is higher than that which maximizes aggregate utility. The max-min criterion ignores the possibility that a rational agent behind the veil might be willing to risk an outcome worse than the maximum possible transfer under the basic income policy regime. Invoking the max-min criterion to justify redistribution is arbitrary and unnecessary. Furthermore, with a heterogeneous population, the aggregate utility maximizing tax-transfer regime does not correspond to common formulations of allocational efficiency. Researchers studying heterogeneous populations should continue to focus on aggregate utility, making sure that any efficiency concept is rigorously defined in relation.

Appendix A: Proofs

Proof of Proposition 4:
Proof. From the derivative
\[
\frac{dU_i}{da_i} = U_1^i (c^* (a_i), h^* (a_i)) c' (a_i) + U_2^i (c^* (a_i), h^* (a_i)) h' (a_i)
\]
and the household optimization condition (3), one can derive the following.
\[
\frac{dU_i}{da_i} = U_1^i (c^* (a_i), h^* (a_i)) h (a_i) (1 - \tau)
\]
The above expression is non-negative so \(U_i\) is non-decreasing in \(a_i\) for any \(a_i > \tilde{a}\). For \(a_i \leq \tilde{a}\), household do not work and consume only the transfer, so \(\frac{dU_i}{da_i} = 0\) for them. ■

Proof of Proposition 5:
Proof. Let the level of productivity \(\tilde{a}\) be such that \(c(\tilde{a}) = Y\). From the equilibrium condition above, we can write
\[
\int_{\tilde{a}}^\infty [Y - c(a_i)] dF (a_i) + \int_{\tilde{a}}^\infty [Y - c(a_i)] dF (a_i) = \int_{\tilde{a}}^\infty [Y - c(a_i)] dF (a_i) = 0
\]
so that the first term is positive and the second term is negative. Since marginal utility \(U_1^i\) is decreasing in \(a_i\),
\[
\int_{\tilde{a}}^\infty U_1^i (c(a_i), h(a_i)) [Y - c(a_i)] dF (a_i) > U_1^i (c(\tilde{a}), h(\tilde{a})) \int_{\tilde{a}}^\infty [Y - c(a_i)] dF (a_i)
\]
and
\[ \int_{\tilde{a}}^{\infty} U_1^i (c (a_i), h (a_i)) [Y - c (a_i)] dF (a_i) > U_1^i (c (\tilde{a}), h (\tilde{a})) \int_{\tilde{a}}^{\infty} [Y - c (a_i)] dF (a_i). \]

Therefore, adding inequalities and using established identities yields the desired result, noting that \( U_1^i (0, 0) Y F (\tilde{a}) \geq 0. \)

Proof of Lemma 8:

Proof. Differentiate relations (11) and (12), with respect to \( \tau \).

\[ \frac{dU_e}{d\tau} \big|_{\tau=0} = U_1^i (0, 0) Y F (\tilde{a}) + \int_{\tilde{a}}^{\infty} U_1^i (c (a_i), h (a_i)) [Y - c (a_i)] dF (a_i) \]

\[ = U_1^i (0, 0) Y F (\tilde{a}) + \int_{\tilde{a}}^{\infty} U_1^i (c (a_i), h (a_i)) [Y - c (a_i)] dF (a_i) \]

\[ > U_1^i (0, 0) Y F (\tilde{a}) + U_1^i (c (\tilde{a}), h (\tilde{a})) \int_{\tilde{a}}^{\infty} [Y - c (a_i)] dF (a_i) \]

\[ = U_1^i (0, 0) Y F (\tilde{a}) + U_1^i (c (\tilde{a}), h (\tilde{a})) \int_{\tilde{a}}^{\infty} [Y - c (a_i)] dF (a_i) > 0. \]

Proof of Proposition 9:

Proof. Evaluation equations (11) and (12) at \( \tau = 0 \) gives \( Y = \left( \frac{1}{\psi + 1} \right) \mu_1 \) and \( \tilde{a} = 0. \) Hence, the
expression (6) for \( \frac{dU^e}{d\tau} \rvert_{\tau=0} \) evaluates to

\[
\frac{dU^e}{d\tau} \rvert_{\tau=0} = u'(0) YF(0) + \int_0^\infty \frac{\psi + 1}{a_i \left( \frac{1}{\psi + 1} \right)} \mu_1 - \frac{a_i}{\psi + 1} \right] dF(a_i)
\]

Since, \( F(0) = 0 \), the term \( u'(0) YF(0) > 0 \), so \( \mu_1 \mu_{-1} > 1 \) guarantees that \( \frac{dU^e}{d\tau} \rvert_{\tau=0} > 0 \).

**Appendix B: Functional forms for the distribution of productivity**

Under minimal parameter restrictions, the condition for welfare improving redistribution in Proposition 9 is met for commonly used functional forms for modeling the distribution of wages and incomes. All of the parameter restrictions are necessary for these distributions to be single peaked on positive values, which are standard stylized facts for income distributions.

**Corollary 10** Let the probability density \( f(a)^\sim(0, \infty) \) have \( r \)th moment \( \mu_r \).

i) For the gamma distribution \( f(a, k, \theta) = \frac{a^{k-1} \exp\left(-\frac{a}{\theta}\right)}{\theta^k \Gamma(k)} \), the condition \( \mu_1 \mu_{-1} > 1 \) is equivalent to \( k > 1 \).

ii) For the lognormal distribution \( f(a, \mu, \sigma) = \exp\left(-\frac{(\ln a - \mu)^2}{2\sigma^2}\right) \frac{1}{\alpha \sigma \sqrt{2\pi}} \), the condition \( \mu_1 \mu_{-1} > 1 \) is equivalent to \( \sigma > 0 \).

iii) For the Weibull distribution \( f(a, \lambda, j) = \frac{j a^{j-1} \exp\left(-\frac{a}{\lambda}\right)}{\lambda^j} \), the condition \( \mu_1 \mu_{-1} > 1 \) is equivalent to \( j > 1 \).

Considerations of standard income/wage distribution and measures of inequality demonstrate that the condition in Proposition 9 is met for any reasonable population.

**References**


Harsanyi, John 1955 Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility,

Jackson, Shirley 1948. The Lottery. in the New Yorker (June 26).


Piketty, Thomas and Saez, Emmanuel 2013 "Optimal Labor Income Taxation" with Thomas Piketty, in the n, Volume 5, 391-474

Pinovský, Maxim and Sala-i-Martin, Xavier 2009 Parametric Estimations of the World Distribution of Income, NBER Working Paper 15433


Werning, Ivan 2007 Pareto Efficient Income Taxation, working paper.