Price Level Targeting with Heterogeneous Expectations: A Little Dab Will Do Ya!

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Abstract

Commitment to price level targeting under rational expectations improves policy outcomes due to the influence on expectations but is not time consistent. Here, public agents can switch between forecasting strategies based on commitment and discretion, according to the learning dynamic of Brown, von Neumann and Nash (1951). A persistent fraction of agents use the discretionary forecasting strategy, and, therefore, a partial degree of commitment is optimal. The quantitative benefit of the optimal policy compared to discretion depends on policymaker preferences.

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JEL classification: E52, E31, D84

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1 Introduction

Commitment to history dependent interest rate rules, where the monetary policymaker targets the level of the price, has a stabilizing effect on public expectations, Woodford (2003). Evans and Honkapohja (2004) show that a class of such rules that respond directly to public expectations have desirable properties of determinacy and expectational stability. However, Waters (2012) demonstrates that such commitment can lead to poor outcomes from a policy perspective, when agents form expectations with adaptive rules such as least squared learning.

All of the above work assumes that the public has a representative forecasting strategy, even though the policy regime is not time consistent. The present work introduces heterogeneous forecasts, where agents recognize the temptations the policymaker faces and can choose forecasts based on commitment or discretion policies. The dynamic of Brown, von Neumann and Nash (1951) describes how agents switch between forecasting strategies based on past performance.

Even if the policymaker follows the commitment strategy, some agents have incentive to adopt the forecasting strategy based on discretion. For some realizations of the model, the discretionary forecasting strategy outperforms the forecasting strategy based on commitment. Further, more agents adopting the discretionary strategy directly affects the endogenous variables, which tends to improve the performance of that strategy.

If some agents form expectations according to discretion, the effect of the commitment policy on expectations in mitigated and the argument in favor of that policy is questionable. Results of simulations allowing for heterogeneous expectations and varying levels of commitment by the policymaker demonstrate the optimality of an intermediate level of commitment, compared to the result with a representative rational expectation we refer to as full commitment. Further, the qualitative implications for policy depend on the preferences of the policymaker. A hawkish policymaker, meaning one whose primary concern is inflation, can achieve a significant improvement over discretion by adopting the intermediate level of commitment. However, a less hawkish policymaker achieves only a small improvement.

The level of commitment corresponds to the degree to which the policymaker affects the price level. Given a one-time supply shock, under full commitment, the policymaker would act to return the price level all the way to its original target path. Under partial commitment, the policymaker makes a partial adjustment to the price level.

The idea that the optimality of full commitment is sensitive to the modeling environment is not new. As noted, Waters (2012) has related, though not identical, findings for a representative public agent using various adaptive learning mechanisms. Woodford (2003, section 7.2.1) notes that the result may be fragile
if the policymaker has a preference for smoothing interest rates or if there is a possibility of the zero lower bound binding. The present work has similar conclusions but for a different reason, the possibility that not all agents have the same forecast.

Other studies have examined the New Keynesian model of monetary policy under heterogeneous expectations. Branch and McGough (2009) study a model where fixed fractions of agents use different information when forming expectations. Branch and McGough (2010) use the multinomial logit dynamic to describe switching between perfect foresight and adaptive forecasts. In Davis (2012) agents switch between forecasts based on different beliefs about the policymaker’s inflation target according a Bayesian learning dynamic. The novel aspects of this paper are the choice of forecasting strategies, based on commitment and discretion, and the choice of the evolutionary dynamic.

The dynamic of Brown, von Neumann and Nash (1951) is particularly appropriate for the present application due to its behavior in and around states where one strategy is eliminated. We should allow for the possibility that all agents use a forecasting strategy based on the actual policy in use. The BNN dynamic has the property of positive correlation, under which strategies that perform well always gain adherents. Further, it has the property of inventiveness, meaning the strategies can gain adherents even if it has none.

2 The Model

The following New Keynesian model has become standard for the study of interest rate rules in monetary policy. It has micro foundations described in Woodford (2003) including price stickiness that allows the policymaker to play a stabilizing role in the economy.

\[
\begin{align*}
    x_t & = -\varphi (i_t - E_t^t \pi_{t+1}) + E_t x_{t+1} + g_t \\
    \pi_t & = \lambda x_t + \beta E_t \pi_{t+1} + u_t
\end{align*}
\]

These equations are expectations augmented IS and Phillip’s curve relationships. The variables \(x_t\) and \(\pi_t\) are the deviations of output and inflation from their target values. The policymaker controls the nominal interest rate \(i_t\). The parameters \(\varphi, \lambda, \beta\) are all positive and the discount rate \(\beta\) is such that \(\beta < 1\). The
stochastic terms \( g_t \) and \( u_t \) both have autoregressive structure.

\[
\begin{pmatrix}
g_t \\
u_t
\end{pmatrix} = F \begin{pmatrix}
\tilde{g}_{t-1} \\
u_{t-1}
\end{pmatrix} + \begin{pmatrix}
\bar{g}_t + \bar{w}_t \\
\bar{u}_t
\end{pmatrix}
\]

(3)

for \( F = \begin{pmatrix}
\mu & 0 \\
0 & \rho
\end{pmatrix} \)

The parameters \( \mu \) and \( \rho \) lie in the interval (-1, 1), and the shocks \( \tilde{u}_t, \bar{w}_t \) and \( \tilde{g}_t \) and are iid with standard deviations \( \sigma_u, \sigma_w \) and \( \sigma_g \), respectively. The structure of \( g_t \) follows McCallum and Nelson (2004) who decompose this term into a preference shock \( \tilde{g}_t \) and an AR(1) process \( \tilde{g}_t = \mu \tilde{g}_{t-1} + \bar{w}_t \) that accounts for uncertainty about the evolution of the natural rate that enters into the forecast error for output in the IS equation (1) and represents the portion of the demand shocks about which the public has some information to use for forecasting.

The policymaker sets the nominal interest rate to stabilize the endogenous variables. Formally the task is to set \( i_t \) to minimize the loss function

\[
L = E_t \sum_{s=0}^{\infty} \beta^s (\pi_t^{2+s} + \alpha x_t^{2+s}),
\]

(4)

assuming rational expectations for the following derivation. The parameter \( \alpha \) measures the relative importance of output and inflation stabilization for the policymaker, the value \( \alpha = 0 \) corresponding to pure inflation targeting. Minimizing the loss function\(^1\) over \( x_{t+s} \) and \( \pi_{t+s} \) under the constraint of the Phillip’s curve (2) yields the following first order conditions.

\[
E_t (2\alpha x_{t+s} + \lambda \omega_{t+s}) = 0 \quad \text{for } s = 0, 1, 2, 3...
\]

(5)

\[
E_t (2\pi_{t+s} + \omega_{t+s-1} - \omega_{t+s}) = 0 \quad \text{for } s = 1, 2, 3...
\]

(6)

\[
2\pi_t - \omega_t = 0
\]

(7)

The variable \( \omega_{t+s} \) is the Lagrange multiplier on the constraint for each \( s = 0, 1, 2, 3... \). Optimal policy in time \( t \) is governed by (7) but policy in succeeding periods is determined by the above condition (6). The time consistency problem is evident since, when the policymaker solves the problem in the period \( t + 1 \), the

\(^1\) The present approach follows Evans and Honkapohja (2004), Woodford (1999b) and Clarida, Gali and Gertler (1999).
policy given by (7) will be different than the policy prescribed by (6) in the period $t$ solution.

To find the condition for policy under discretion, the policymaker adopts the optimal policy each period, solving out $\omega_t$ for $s = 0$ from the first order conditions (5) and (7).

$$\lambda \pi_t + \alpha x_t = 0$$  \hspace{1cm} (8)

Under discretion, the policymaker takes expectations to be fixed and ignores (6), see Clarida, Gali and Gertler (1999) for a detailed discussion. However, if the policymaker has a "timeless perspective" (Woodford 1999b) he or she uses the policy that would have been prescribed in past periods, ignoring condition (7), using (5) and (6) for $s = 0$. In this case, which we refer to as full commitment, the policymaker acts to influence future expectations yielding the condition

$$\lambda \pi_t + \alpha (x_t - x_{t-1}) = 0.$$  \hspace{1cm} (9)

Policy under commitment differs from discretion in its response to the previous period’s output gap. Such a history dependent policy uses the policymaker’s influence on public expectations to minimize fluctuations.

There are several reasons to question full commitment, meaning the policymaker should not be restricted to these extreme cases. First, some agents may not believe the policymaker will commit to a time inconsistent policy. A related issue is that gains to commitment depend on a forward looking public, so if the public is using a learning mechanism to form expectations, the policymaker may not want to fully commit. Waters (2005, 2009) shows that discretion is superior to commitment under, when the public forecast is formed using a learning rule.

Hence, we consider the following general condition that allows for varying degrees of commitment.

$$\lambda \pi_t + \alpha (x_t - \kappa x_{t-1}) = 0.$$  \hspace{1cm} (10)

The condition under discretion is a special case of (10) where $\kappa = 0$, and full commitment is equivalent to setting $\kappa = 1$. Any intermediate setting for $\kappa$ such that $0 < \kappa < 1$ is referred to as partial commitment.$^2$

### 3 Expectations Based Interest Rate Rules

Evans and Honkapohja (2003, 2006) advocate for monetary policy to be conducted with interest rate rules that respond explicitly to public expectations. Such rules have desirable properties of determinacy and

$^2$Blake (2001) and McCallum and Jensen (2002) advocated for policy equivalent to setting $\kappa = \beta$. 

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expectational stability for any level of commitment, see Waters (2009). Given the model above, we can
derive the expectations based interest rate rule for any level of commitment $\kappa > 0$.

Using (10) to substitute for inflation in the Phillips curve (2) gives

$$x_t = \lambda (\lambda^2 + \alpha)^{-1} (\alpha \kappa x_{t-1} - \beta E_t \pi_{t+1} - u_t).$$

Substituting out $x_t$ in the IS equation with the above expression yields the interest rate rule

$$i_t = \delta_L x_{t-1} + \delta_\pi E_t^* \pi_{t+1} + \delta_x E_t^* x_{t+1} + \delta_g g_t + \delta_u u_t \quad (11)$$

where the parameters are

$$\delta_L = -\varphi^{-1} (\lambda^2 + \alpha)^{-1} \alpha \kappa, \quad \delta_\pi = 1 + \varphi^{-1} (\lambda^2 + \alpha)^{-1} \lambda \beta$$

$$\delta_x = \varphi^{-1}, \quad \delta_g = \varphi^{-1}, \quad \delta_u = \varphi^{-1} (\lambda^2 + \alpha)^{-1} \lambda.$$

Note that the extent to which the interest rate responds to lagged output, shown by $\delta_L$, depends on $\kappa$, but
the other parameters in the rule do not. Under discretion, $\kappa$ is zero and $i_t$ is unaffected by $x_{t-1}$, but for any
other value, the interest rate responds directly to $x_{t-1}$. The greater the degree of commitment, the greater
the magnitude of the response to the lagged output gap.

4 RE Equilibria

One can solve for the unique minimum state variables\(^3\) rational expectations equilibrium (REE) for a given
$\kappa$ in the general condition (10). Postulating solutions of the form

$$x_t = b_x (\kappa) x_{t-1} + c_x (\kappa) u_t$$

$$\pi_t = b_\pi (\kappa) x_{t-1} + c_\pi (\kappa) u_t \quad (12)$$

implies that $E_t (\pi_{t+1} | \Omega_t) = b_\pi (\kappa) (b_x (\kappa) x_{t-1} + c_x (\kappa) u_t) + c_\pi (\kappa) \rho u_t$. As in a number of related studies
including Evans and Honkapohja (2006), expectations in time $t$ are based on lagged endogenous variables
and contemporaneous shocks, meaning that for $z_t = (x_t, \pi_t, i_t)$ and $\Theta_t = (u_t, g_t)$, the information set is such
that $\Omega_t = \{\Theta_t, \Theta_{t-1}, z_{t-1}, \Theta_{t-1}, z_{t-2}, \Theta_{t-2}, \ldots\}$. This convention concerning the information set hold for

\(^3\)See McCallum (1983, 1997).
the succeeding analysis and is suppressed.

Using the method of undetermined coefficients with (10) and (2) yields

\[ b_{\pi} (\kappa) = \lambda b_x (\kappa) + \beta b_{\pi} (\kappa) b_x (\kappa) \]  
\[ \lambda b_{\pi} (\kappa) = \alpha (\kappa - b_x (\kappa)) \]  
\[ c_{\pi} (\kappa) = \lambda c_x (\kappa) + \beta (b_{\pi} (\kappa) c_x (\kappa) + c_{\pi} (\kappa) \rho) + 1 \]  
\[ \lambda c_{\pi} (\kappa) = -\alpha c_x (\kappa) . \]

Combining the first two equations from (13) gives us

\[ \beta b_x (\kappa)^2 - \left( \frac{\lambda^2}{\alpha} + \beta \kappa + 1 \right) b_x (\kappa) + \kappa = 0 \]  

The larger root of the quadratic equation (14) is always such that \( b_x (\kappa) > 1 \) so the smaller root is the potentially non-explosive solution for \( b_x \) in (12) and is the focus throughout the paper. Given that choice of solution for \( b_x (\kappa) \), the four equations above fully characterize the solution of interest.

Note that under discretion, it is the case that \( b_x (0) = b_{\pi} (0) = 0 \), which corresponds to the fact that the minimum state variables solution in this case depends only on the supply shock \( u_t \), as in Evans and Honkapohja (2003). Further, for \( \kappa > 0 \), the relevant solutions for \( b_x \) and \( b_{\pi} \) are strictly positive. Hence, public expectations respond to lagged output only when the policymaker does, demonstrating the policymaker’s influence on expectations under commitment. The solutions for the coefficients under full commitment for \( \kappa = 1 \) correspond to those in Evans and Honkapohja (2006).

Commitment implies that the policymaker is responding to the price level \( p_t \), not just inflation. Under partial commitment, the policymaker also responds to the price level but is not concerned with returning the price all the way to it original level as he would be under full commitment.

Impulse responses in Figure 1 demonstrate the qualitative differences in the policies. Three graphs represent the responses to a one time supply shock \( u_0 = 1 \) of the inflation and output gaps and the price level given the starting value \( p_{-1} = 0 \). Under discretion, the output and inflation gaps are eliminated in the next period, and the price level is allowed to remain at its new level. Under full commitment, however, the output gap and the price level are returned to zero slowly. The original price level matters, one way to express the history dependence of commitment.
Under partial commitment, the policymaker acts to counter the effect of the supply shock on the price level, but not to the original level. The higher the degree of commitment the closer the long run target for the price level is to zero. This idea is expressed formally as follows.

**Proposition 1** For the model given by (1) and (2) with the minimum state variables solution under rational expectations, let the initial price level be zero, \( p_{-1} = 0 \), and the coefficients \( b_x \) and \( c_\pi \) be such that \( 0 < b_x < 1 \), and \( c_\pi > 0 \). For a one time supply shock \( u_o = 1 \), the long run price target under the interest rate rule (11) is

\[
\bar{p} = \lim_{t \to \infty} p_t = c_\pi \left( \frac{1 - \kappa}{1 - b_x} \right).
\]
Hence, the long run price target is inversely related to the level of commitment.

\[ \frac{d\bar{p}}{d\kappa} < 0 \]

5 Forecasting strategies

Public agents form expectations based on both the discretionary and commitment solutions. Even when policy is set under commitment, the public is aware that the policymaker has incentive to adopt the discretionary policy in any period. The available strategies are the rational forecasts under two different assumptions about policy. The policymaker announces his level of commitment \( \kappa \), which determines the coefficient \( \delta_{-1} \) on lagged output in the interest rate rule (11). One forecasting strategy, with expectation denoted \( E^c \), takes this level of commitment as given, and the other assumes the policy is set by discretion, which is equivalent to \( \kappa = 0 \) and has expectation denoted \( E^d \). Discretionary expectations are determined by the rational expectations solutions (12) for \( \kappa = 0 \), recalling that \( b_x (0) = b_\pi (0) = 0 \).

\[ E^d_{t-1} x_t = c_x (0) \rho u_{t-1} \]  
\[ E^d_{t-1} \pi_t = b_\pi (\kappa) b_x (\kappa) x_{t-1} + c_\pi (\kappa) (b_\pi (\kappa) + \rho) u_{t-1} \]

Agents know the policymaker’s degree of commitment \( \kappa \), so commitment expectations \( E^c \) are also given by (12) substituting for \( E^c_{t-1} x_{t-1} = b_x (\kappa) x_{t-2} + c_x (\kappa) \rho u_{t-1} \) as follows.

\[ E^c_{t-1} x_t = b_x (\kappa)^2 x_{t-1} + c_x (\kappa) (b_x (\kappa) + \rho) u_{t-1} \]  
\[ E^c_{t-1} \pi_t = b_\pi (\kappa) b_x (\kappa) x_{t-1} + c_\pi (\kappa) (b_\pi (\kappa) + \rho) u_{t-1} \]

In the presence of heterogeneous forecasts, the expectations in the IS (1), Phillips Curve (2) and policy rule (11) relations are an aggregation of the two available forecasts weighted according to popularity. For two strategies \( k = d, c \), let the fraction of the population using the discretionary forecasting strategy be \( q_{d,t} \) be such that \( q_{d,t} = q_t \). Similarly, the fraction of followers of the commitment forecasting strategy is such that \( q_{c,t} = 1 - q_t \).

\[ E_t x_{t+1} = q_t E^d_t x_{t+1} + (1 - q_t) E^c_t x_{t+1} \]
Given $q_t$ using the IS relation (1), the Phillips Curve (2) the policy rule (11) equations, the forecasts (15), (16), (17) and (18), and the aggregate expectations (19), the model can be represented as follows.

\[ x_t = f(q_t)x_{t-1} - \lambda h(q_t)u_t \]  
\[ \pi_t = \alpha g(q_t)x_{t-1} + \alpha h(q_t)u_t \]  

for the functions

\[ f(q_t) = \alpha \kappa - \beta \lambda b \pi (1 - q_t) \]  
\[ g(q_t) = \lambda \kappa + \beta b \pi (1 - q_t) \]  
\[ h(q_t) = 1 - \left( \frac{\alpha \beta \rho}{\lambda^2 + \alpha} \right) q_t + \beta (b \pi + \rho) c \pi (1 - q_t) \]

### 6 Evolution of Expectations

The evolutionary dynamic of Brown, von Neumann and Nash (1950) describes how agents switch between forecasting strategies based on the past performance and, therefore, the evolution of $q_t$. Performance of the forecasting strategies is determined by the squared errors of the forecasts for output and inflation weighted the same way as the in policymaker’s loss function, which is derived from an approximation of a welfare function, see Woodford (2003). For strategy $k$, let the forecast of variable $z_t$ given $\Omega_t$ be $E^k_{t-1}z_t$, so the payoff in time $t$ to strategy $k$ is

\[ U^k_k = - \left[ (\pi_t - E^k_{t-1}\pi_t)^2 + \alpha (x_t - E^k_{t-1}x_t)^2 \right] / \bar{U} \]  

where $\bar{U}$ is a constant scaling term.

Let the fraction of followers of strategy $k$ in time $t$ be $q_{k,t}$, and the number of available strategies be $H$. The $\delta$–BNN dynamic depends on the excess payoffs $\hat{U}_{k,t}$, which is the difference between the payoff $U_{k,t}$ and the population average $\bar{U}_t = \sum_{h=1}^{H} q_{h,t} U_{h,t}$. In the following dynamic, the notation $[\cdot]_+$ is such that $[Y]_+ = \max(0,Y)$.

\[ q_{k,t+1} = \frac{q_{k,t} + \left[ \hat{U}_{k,t} \right]^\delta}{1 + \sum_{h=1}^{H} \left[ \hat{U}_{h,t} \right]^\delta} \]  

For sufficiently large excess payoffs – specifically, to the right of the inflection point of $U^\alpha$ – the parameter
alpha represents the aggressiveness of agents in switching to better performing strategies. For larger $\alpha$, there is extra weight on larger excess payoffs and a larger portion of agent switch to those strategies.

The $\alpha$-BNN dynamic is particularly appropriate for the present application because it satisfies the properties of inventiveness and positive correlation, which have implications for behavior near the edges of the dynamic where $q_k = 0$. When policy is given by commitment, we should allow for the possibility that the discretionary forecasting strategy is eliminated. Positive correlation is a weak monotonicity condition ensuring that, in aggregate, strategies that perform poorly have a decreasing fraction of followers\(^4\). The commonly used\(^5\) multinomial logit dynamic does not satisfy this condition. Inventiveness means that strategies that have no followers can gain some. Imitative dynamics such as the replicator do not have this property\(^6\). See Sandholm (2011) and Waters (2010) for a detailed discussion.

Given the series of shocks $\{g_t\}, \{u_t\}$ and starting values for the endogenous variables $(x_t, \pi_t, i_t, q_t)$, the evolution of the model is determined by the equations for output and inflation (20) and (21), the expectations (15), (16), (17) and (18) along with the payoffs for both strategies given by (22) and the evolutionary dynamic (23).

## 7 Motivation for Heterogeneity

Even when policy is set according to commitment, there is incentive to use forecasts based on discretion. The discretionary forecasting strategy is self-fulfilling in the sense that the greater the fraction of agents using the strategy, the better it performs. Furthermore, the relative performance of the commitment and discretionary forecasting strategies depends on the state of the economy, in particular the correlation between information on the output gap and the supply shock available to the agents when making forecasts. For the following exercises, let the parameters take the values as in McCallum and Nelson (2004).

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\phi$</th>
<th>$\lambda$</th>
<th>$\rho$</th>
<th>$\sigma_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>0.4</td>
<td>0.05</td>
<td>0.8</td>
<td>0.005</td>
</tr>
</tbody>
</table>

Figure 2 shows the expected payoffs $E_{t-1}U^d_t$ and $E_{t-1}U^c_t$ to the forecasting strategies under discretion and commitment for varying values of the fraction using the forecast based on discretion $q_t$, assuming that it is fixed over time $q = q_t = q_{t-1} = \ldots$. The payoffs are calculated for given values of the information available

\(^4\)Formally, the condition for positive correlation is $\sum_{h=1}^H (q_{h,t+1} - q_{h,t}) \hat{U}_{h,t} > 0$ unless $\hat{U}_{h,t} = 0$ for all $h$.

\(^5\)Notable papers using the multinomial logit dynamic include Brock and Hommes (1997) among many others.

\(^6\)Parke and Waters (2007) use a generalized version of the replicator and introduce a small fraction of the population using a strategy when it is close to elimination. This is similar to the concept of drift described in Samuelson ( ).
to the agents \( x_{t-2} = 0.002 \) and \( u_{t-1} = 0.01 \) with parameter settings \( \kappa = 1.0 \), where the policymaker fully commits, and \( \alpha = 0.25 \), where there is equal emphasis on output and inflation stabilization\(^7\). The precise shape of the curve depends on the given values of \( x_{t-2} \) and \( u_{t-1} \), but some general observations can be made. Both payoffs are increasing in \( q \) in a neighborhood of \( q = 0 \). Also, neither strategy dominates for all values of \( q \). It appears that the commitment forecasting strategy is superior when everyone is using it \( (q = 0) \), but this need not be true in general.

**Figure 2**

![Figure 2](image1)

**Figure 3**

Payoff differences \( E_{t-1} \left( U_d^t - U_c^t \right) \)

![Figure 3](image2)

To further examine this issue, we examine the difference in the expected payoffs \( E_{t-1} \left( U_d^t - U_c^t \right) \) as a

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\(^7\)This interpretation is appropriate since inflation is commonly measured as an annual percentage change and the estimated parameter are based on quarterly data.
function of the supply shock $u_{t-1}$ for a given value of output gap $x_{t-2}$. The fraction $q$ is set to zero to examine the possible incentives agents have to deviate from the commitment forecast. For $x_{t-2} = 0.0$ in Figure 3, forecasting under commitment is always superior as the payoff difference is negative for any value of $u_{t-1}$. This result is natural since zero is the timeless perspective expected value of $x_{t-2}$, meaning the expected value infinitely far in the future. The superiority of the commitment forecasting strategy comes about, since that forecast is precisely the rational expectation under full commitment by the policymaker.

However, the output gap varies over time and for a non-zero value of $x_{t-2} = 0.001$, there is a range for $u_{t-1}$ where the discretionary forecasting strategy is superior. Note that the range is skewed to the right so the correlation between $x_{t-2}$ and $u_{t-1}$ is critical. For the value $x_{t-2} = -0.001$, the graph in Figure 3 would be reflected around the vertical axis. Hence, for some realizations, public agents have incentive to switch to the discretionary forecasting strategy.

Figure 4 characterizes the combinations of the output gap and supply shock determining the relative performance of the strategies. For the first two graphs, all agents are using the commitment forecasting strategy so $q = 0$. The area in green shows where commitment is slightly superior, i.e. the payoff difference $E_{t-1} (U_t^d - U_t^c) < -z$ for $z = 10^{-8}$. In the following figure the threshold $z$ is set to 0 and the plane is divided by two line passing through the origin. The possible realizations where some agents would adopt the discretionary forecasting strategy are common. The third graphs shows the results of the same exercise for $q = 0.25$. More agents adopting the discretionary strategy only increases the chance that the discretionary strategy has a superior performance. A limitation of these results is that $x_{t-2}$ and $u_{t-1}$ are correlated if there is any persistence ($\rho > 0$) in the supply shock. However, the results when $\rho$ is set to zero are qualitatively similar.
Clearly, agents have incentive to adopt the discretionary forecasting strategy for some realizations in the model. The next issues that arise are the long run choices of forecasting strategies in the population and the implications for the optimal degree of commitment by the policymaker. These questions are addressed in the succeeding simulation results.

The fraction of the public using the forecast based on full commitment fluctuates around an interior value in the interval $(0, 1)$. Figure 5 shows the evolution of the fraction $q_t$ for a sample simulation using the same parameter values as in the previous section. All the endogenous variables including $q_0$ are initialized at zero. To properly scale the payoff, we calculate the expected payoff to discretion from a timeless perspective. The scaling parameter $\tilde{U}$ is that expected payoff times $10^5$ so that the excess payoff are ensured to be large enough to have a noticeable impact on $q_t$ and so that changes in the parameter $\delta$ in the evolutionary dynamic (23) have intuitive effects. For the following analysis, the parameter $\delta$ is set to one, so scaling is not a major issue.

8 Optimal Policy with Heterogeneity

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\(^8^{The expected payoff to strategy k from a timeless perspective means } \lim_{t \to \infty} E_t U_t^k \)
Though it takes time for the \( q_t \) to rise above its initial value of zero, it eventually fluctuates around approximately 0.2. That the dynamic (23) allows for such behavior is an illustration of the inventiveness property.

The population state where all agents use the forecasting strategy based on commitment by the policymaker, \( q = 0 \), cannot be stable. For non-zero value of the output gap, there are many realizations of the supply shock and output gap such that the correlation between the two lead the commitment forecasting strategy to underperform. Further, as more agents adopt the discretionary forecasting strategy, its performance improves leading to greater adoption. Since, the discretionary forecasting strategy is at odds with actual policy, the state where \( q = 1 \) is not stable either, so the variable \( q_t \) is destined to fluctuate with the other endogenous variables.

Given the inherent heterogeneity in the choices of forecasting strategies by the public, the optimality of full commitment by the policymaker becomes questionable. Figure 6 shows simulation results (4) for varying levels of commitment by the policymaker \( \kappa \). Values of losses from simulations of the model, detailed at the end of section 6, are averages of a minimum of 5000 runs of 3000 time periods with 500 period for initialization. The simulations are run for two separate choices of \( \alpha \), the setting \( \alpha = 0.25 \) where the policymaker places equal emphasis on output and inflation stabilization, and the setting \( \alpha = 0.01 \), indicating a policymaker who is very hawkish about inflation. To compare the two cases, the values reported in the figure are the losses for varying levels of commitment \( \kappa \) as a ratio with the loss under discretion where \( \kappa = 0 \).
For both cases, some degree of commitment improves policy performance in comparison with discretion, but full commitment ($\kappa = 1.0$) is not optimal. In the case where $\alpha = 0.25$, the outcome under full commitment is far worse than under discretion. A similar degree of partial commitment is optimal in both cases. As shown in Table 1 in the Appendix, for $\alpha = 0.25$, the optimal (loss minimizing) level of commitment is $\kappa = 0.725$ and for $\alpha = 0.01$ the optimal level is $\kappa = 0.775$. Both correspond to the partial commitment shown in Figure 1, where the policymakers acts to counter the effect of the supply shock on the price level but not to the extent that the price level returns to its original level.

There are qualitative differences in the policy implications for the two cases of policymaker preferences. For $\alpha = 0.25$, the benefits of any degree of commitment are modest at best, and the optimal level produces a loss only 2% better than discretion. Furthermore, excessive commitment where $\kappa$ is larger than optimal produces a very poor outcome, which represents an additional risk if there is parameter or model uncertainty. For the hawkish policymaker ($\alpha = 0.01$), however, the optimal level of commitment is a 28% improvement and the losses are symmetric around this choice of $\kappa$. This result matches the finding of Waters (2009) for a representative agent using least squares learning, though the optimal level of commitment for the hawkish policymaker is higher in that environment.

One might question the degree to which these results are driven by the fluctuations in the fraction $q_t$. To examine this issue, we conduct the above exercise for a fixed $\bar{\pi}_t$, where its value is given by the long run average in the above simulations. Thought the losses are lower for any given choice of parameters, the ratios
of the losses with the discretionary loss are very similar to those in Figure 6 and the policy implications are unchanged.

Another concern is that in the presence of heterogeneity, neither forecasting strategy satisfies rational expectations. The rational forecast would be an average of the discretionary and commitment forecasts weighted according to $q$. Preliminary results for simulations that include such a "reflective" forecast show that the implications for policy are unchanged.

9 Conclusion

Given that price level targeting is not a time consistent policy, it is reasonable to assume that some public agents will form forecasts assuming discretion. The analysis here demonstrates that they will receive ample substantiation of their choice. Since persistent heterogeneity in public expectations is to be expected, full commitment, meaning the policymaker completely offsets the effect of a supply shock on the price level, is not optimal. Some degree of price level targeting is advisable, though the potential benefits to the policy and costs of overdoing it are sensitive to the preferences of the policymaker.

From a broad perspective, the present work offers a warning about results that are dependent on representative agents and/or forecasts. When plausible alternative forecasting strategies are available, for which there is often abundant evidence, conclusions drawn from models with a unique expectation must be treated with care.

References


Table 1

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