Strong Efficiency, Weak Efficiency and Endogenous Rational Bubbles
Mysticism in Asset Markets

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I'd be a good stock market expert.

I'd buy stocks and then go on TV and recommend them so they go up.

What about the fundamentals?

It doesn't get more fundamental than that!

Stock market expert...

...everyone should buy stock in that company. Sell your house if necessary.

Should we worry that the P/E is 900, your track record is terrible and you only recommend stocks you own?

Well, Ron, as you can see from the one-week chart, this stock only goes up.

Buy! Buy!
Strong Efficiency
- present value relation
- questionable due to variance bounds etc.
- forecastable returns and excess variance are equivalent(?!)

Weak Efficiency
- prices/returns are unforecastable
- appealing theoretically
- evidence is mixed

Rational bubbles
- extraneous random walk data matters
- satisfies weak version
- violates the strong version
- how do they form?
Forecastable Returns: A Simple Test

\[ r_{t+k} = \beta_0 + \beta_1 (p_t - d_t) + \varepsilon_t \]

\( k \) is the forecast horizon

Null: \( \beta_1 = 0 \)

Typical Results:
- \( \hat{\beta}_1 < 0 \)
- marginal t-stats
- low \( R^2 \)

Issues:
- persistence in \( d_t \)
- estimates biased away from 0

\textit{Weak evidence for weak efficiency}
Endogenous Rational Bubbles - Key Elements

- Heterogeneous Forecasts
  - fundamental

- Evolutionary Game Theory

- Explains
  - Excess variance
  - GARCH
Endogenous Rational Bubbles - Key Elements

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Evolutionary Game Theory
- endogenous choice of forecasts
- forecast errors

Explains
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Evolutionary Game Theory
- endogenous choice of forecasts
- forecast errors
- aggressiveness

Explains
- Excess variance
- GARCH
$p_t = \alpha p_{t+1} + d_t$

$p_t$ - asset price
$\alpha$ – discount factor
$d_t$ - dividends
Assuming: (Brock and Hommes 1998)

- mean – variance optimization
- common estimate of variance
- constant supply of the asset

\[ p_t = \sum_{j=1}^{n} \alpha x_{j,t} e_{j,t} + d_t - C \]

- \( e_{j,t} \) - forecast using using strategy \( j \)
- \( x_{j,t} \) - fraction using strategy \( j \)
- \( C \) - constant risk premium
Homogeneous RE solutions

Fundamental Solution

\[ p_t^* = d_t + \sum_{j=1}^{\infty} \alpha j d_{t+j} \]

General Solution

\[ p_t = p_t^* + m_t \]

where \( m_t = m_{t-1} + \eta_t \)
Forecasts

Fundamentalism

\[ e_{2,t} = E(p_{t+1}^*) \]

Mysticism

\[ e_{3,t} = E(p_{t+1}^*) + \alpha^{-t-1} m_t \]

Reflectivism \( e_{1,t} \)

combination of forecasts
\[
e_{1,t} = E(p^*_{t+1}) + \alpha^{-t-1} n_t m_t
\]
for \( n_t = \frac{x_{3,t}}{x_{2,t} + x_{3,t}} \)

- other forecasts weighted according to popularity
- uses "all available" information
- provides an unbiased forecast of the asset price
- “beauty contests”
Realized asset price

\[ p_t = p_t^* + \alpha^{-t} n_t m_t \]

If \( n > 0 \), strong efficiency is violated.
Squared forecast errors:

\[ \pi_{i,t} = - (p_t - e_{i,t-1})^2 \]
Imitative Dynamics

\[ x_{i,t+1} - x_{i,t} = x_{i,t} \frac{w(\pi_{i,t}) - \bar{W}_t}{\bar{W}_t} \]

\[ \bar{W}_t = x_{1,t}w(\pi_{1,t}) + \cdots + x_{n,t}w(\pi_{n,t}) \]

- \( w(\pi) \) is the weighting function
- linear \( w(\pi) \) yields the replicator
- convex \( w(\pi) \) implies more aggressive switching
Exponential weighting

$w(\pi) = e^{\theta^2 \pi}$

- Extra weight on high payoffs
- Higher $\theta$ implies more aggressive switching
- Mysticism can gain a following if
  - agents are sufficiently aggressive
  - shocks are large
  - the martingale has intermediate magnitude
$$d_t = \bar{d} + \rho (d_{t-1} - \bar{d}) + \nu_t$$ calibrated to Shiller’s annual data

Waters (Illinois State University)
Excess Returns

\[ Z_t = p_t + d_t - \alpha^{-1} p_{t-1} \]

\[ Z_t = \bar{Z} + U_t \]

Reflective forecast error:

\[ U_t = (p_t^* - E_{t-1}(p_t^*)) + \alpha^{-t}(n_t m_t - n_{t-1} m_{t-1}) \]
Reflectivism is unbiased:

\[
E_{t-1} U_t = \alpha^{-t} E_{t-1} (\Delta n_t m_{t-1})
\]

if \( n_t \) is not forecastable

Therefore,

\[
E_{t-1} Z_t = \bar{Z}
\]

BUT, persistence in \( n_t \) could affect tests on simulated data
### Excess Volatility

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Ratio of price volatility to volatility implied by the EMH.
A Simple Test

\[ r_{t+k} = \beta_0 + \beta_1 d_t + \varepsilon_t \]

\( k = 2 \) is the forecast horizon

Null: \( \beta_1 = 0 \)

Issues: (in the absence of bubbles)

- persistence in \( d_t \) (AR1)
- estimates of \( \hat{\beta}_1 \) biased away from 0

BUT

a different \( t \) - stat is valid
% with predictable turns, using the standard $t$-statistics for $\beta_1 = 0$ with critical value 1.65

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$\sigma_\eta = \sigma_v x^{1/8, 1/4, 1/2, 1, 2, 4, 8}$

*
Regress excess returns on the innovation to the dividends

\[ r_{t+k} = \beta_0 + \beta_1 (d_t - \rho d_{t-1}) + \epsilon_t \]

- the \( t \)-stat on \( \hat{\beta}_1 \) is valid with different critical values
- \( \rho \) must be known (and \(<1\)
% predictable returns with new critical value -2.0779

\[ \sigma_\eta = \sigma_v x \]

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Theory:
- Forecastability could be an artifact of
  - heterogeneous expectations
  - persistence from learning

Simulations
- qualitatively match the data
- proper tests do not show forecastability
Equivalence of forecastability and excess variance
- depends on linearization around a steady state P/D ratio
- not valid in the presence of mysticism

Arguments against unit roots in the P/D ratio depend on long run behavior

Evidence for weak efficiency does not imply strong efficiency

If mysticism is a reasonable alternative, the methodology for testing forecastability has very low power
Next steps

- greater persistence
- price-dividend ratio as a predictor
  - unknown persistence parameter
  - potential unit root
- local-to-unity asymptotics
  - Campbell & Yogo (2006)
- Bonferroni bounds
$R^2$ statistics for returns forecastability

\[ \sigma_\eta = \sigma_v x \]

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Realized asset price

\[ p_t = p^*_t + \alpha^{-t} n_t m_t \]

Reflective Forecast Error

\[ U_t = p^*_t - E(p^*_t) + \alpha^{-t} (n_t \Delta m_t - \Delta n_{t-1} m_{t-1}) \]

\[ A_t = \alpha^{-t} m_{t-1} \]
Payoffs

Reflective

\[ \pi_{1,t} = -U_t^2 \]

Fundamentalist

\[ \pi_{2,t} = -U_t^2 - 2 n_t U_t A_t - n_t^2 A_t^2 \]

Mystic

\[ \pi_{3,t} = -U_t^2 + 2(1 - n_t) U_t A_t - (1 - n_t)^2 A_t^2 \]
Fitness

Population Average Payoff

$$\bar{\pi}_t = x_{1,t} \pi_{1,t} + x_{2,t} \pi_{2,t} + x_{3,t} \pi_{3,t}$$

Reflective fitness

$$\pi_{1,t} - \bar{\pi}_t = \frac{x_{2,t} x_{3,t}}{x_{2,t} + x_{3,t}} A_{t}^2 > 0$$