Credit Rationing and the Phillips Curve

George A. Waters*
Department of Economics
Campus Box 4200
Illinois State University
Normal, IL 61761-4200

December 10, 2008

Abstract

This paper examines the implications of including credit requirements for firm production according to Boissay (2001) in the micro foundations of the New Classical Phillips curve. Firms require working capital for production and must collateralize their loans. The fraction of firms receiving financing is endogenous, and the interest rate affects the supply decision through two channels, the cost of production and quantity rationing. Furthermore, expected changes in nominal output now enter the linearized aggregate supply relation in a counter-intuitive fashion.

*gawater@ilstu.edu
1 Introduction

The idea that financial variables affect firm production and pricing decisions should not be controversial. Though recent research has focused on credit channels for monetary policy, many of the standard aggregate supply or Phillips curve relations do not include a role for interest rates or financial considerations.

Ravenna and Walsh (2006) develop a micro-founded New Keynesian Phillips Curve where the real interest rate enters into the firm’s decision. The credit channel operates through a cash-in-advance constraint on the firm wage bill, following Christiano and Eichenbaum (1992), that represents price rationing where the cost of financing affects profits. The present paper expands on this work by studying the supply relation that arises in a general equilibrium model where firms have heterogeneous requirements for working capital for production as in Boissay (2001). This approach has price rationing as in Ravenna and Walsh (2006) but also includes quantity rationing where some firms are denied loans. Firms cannot pre-commit to repaying loans so only firms with sufficient collateral get financing, and the fraction of firms that receive financing is endogenously determined.

The resulting linearized aggregate supply relation includes the real interest rate arising from price rationing, as in Ravenna and Walsh (2006), but also includes the fraction of firm receiving financing representing endogenous quantity rationing. Such a variable could be interpreted in terms of flows of funds in the credit market. Furthermore, this fraction depends on the real interest rate, so there is a second credit channel for monetary policy. In addition, expected future nominal output also enters the resulting reduced form supply relation, since agents anticipate future conditions in the market for deposits.

The Phillips Curve has taken many forms. Undergraduate texts now explain Friedman (1968) and Phelp's (1968) objections to the simple inflation-unemployment trade-off, since it implies that a monetary policymaker could keep unemployment permanently below the natural rate. To capture their point that public expectations would respond to such expansionary monetary policy, some researchers such as Sargent and Wallace (1975) suggested the New Classical Phillips Curve, where \( \pi_t \) is inflation, \( Y_t - \bar{Y} \) is the output gap and \( \kappa \) is a positive parameter.

\[
\pi_t = E_{t-1} \pi_t + \kappa (Y_t - \bar{Y})
\]

In this specification, if expectations are rational, then output cannot be kept above its natural rate \( \bar{Y} \). Such a specification is extreme in the sense that output is solely determined by forecast errors and monetary policy has no impact. Alternatively, under Calvo (1983) pricing, firms cannot switch their output prices with positive probability, which leads to the expectation term \( E_{t-1} \pi_t \) being replaced by \( E_t \pi_{t+1} \) in (1). If there are frictions in the price or wage process, monetary policy can have real effects. Forward looking
versions of (1) reflecting such frictions are often referred to as New Keynesian Phillips Curves\(^1\). In order to focus on the effects of credit rationing, the model presented here does not include Calvo pricing, though that is a potential alternative.

The dynamic stochastic general equilibrium model presented here does have a number of standard New Keynesian elements. Intermediate goods producers are monopolistic competitors and produce heterogeneous goods, while there is a single final good. Households face a labor-leisure trade-off and are able to make interest bearing deposits for one period. To avoid the classical dichotomy, there must be some type of price stickiness. Here, a fraction of intermediate goods producers must set their prices one period in advance\(^2\). In a New Keynesian general equilibrium framework, Woodford (2002) shows that this specification for pricing produces a New Classical Phillips curve similar to (1).

The details of production follow Boissay (2001) who introduces the working capital requirement into a real business cycle model. Firms must take loans to produce and have heterogeneous requirements for working capital. Furthermore, as mentioned above, firms must collateralize their loans, and only a fraction of firms get financing so there is credit rationing with respect to the quantity of loans received. Boissay (2001) points out that this model is a type of financial accelerator, since collateral constraints bind more firms as output declines.

The focus on quantity rationing, where some firms do not get loans, as opposed to price rationing, where firms pay a premium due to financial frictions, has support in the empirical literature\(^3\). However, much of the theoretical work on credit channels emphasizes price rationing. Bernanke and Gertler (1989) is a prominent example of a model with price rationing, where costly state verification (monitoring of borrowers) creates an external cost to firm borrowing. Carlstrom and Fuest (1997) study monetary policy in a general equilibrium model with agency costs. The financing premium exists in the credit rationing model in Boissay (2001) as well, but there is a far greater impact on real activity due to the quantity rationing directly attributable to the collateral constraint.

The resulting supply relation includes the elements found in the New Classical Phillips curve (1) with a number of additional features. Like the model of Ravenna and Walsh (2006) the real interest rate now enters the supply decision. In their approach, there is a cash-in-advance constraint requiring firms to hold money to pay wages, so the interest rate affect the cost of production, which is the case in the model in this paper, as well. The presence of quantity rationing opens a second credit channel, since a higher interest rate

---

\(^1\) The output gap is frequently replaced with real marginal cost as in Sbordone (2002).


\(^3\) Kayshap, Lamont and Stein (1994) study the effect of monetary policy on financial flows of funds. See Boissay (2001) for further references.

\(^4\) Kiyotaki and Moore (1997) has a general equilibrium model with quantity rationing, though there is no direct connection with the monetary policy literature. Lamont (1997) has a general equilibrium model including debt overhang.
rate can tighten the collateral restraint, so a smaller fraction of firms receive financing. Furthermore, agents anticipate future credit conditions so an expected increase in nominal output implies a greater demand for savings, lower consumption and, consequently, lower inflation.

2 The model

Following standard New Keynesian approaches, there is nominal stickiness in that monopolistic competitors do not all set prices at the same time. The primary departure of this model from standard approaches is the introduction of a working capital requirement for firms.

2.1 Demand for intermediate goods

Intermediate goods producers are monopolistic competitive and produce differentiated goods \( y_t(i) \) and set prices \( p_t(i) \) in time \( t \). Final goods \( Y_t \) are produced from intermediate goods according to

\[
Y_t = \left( \int_0^1 y_t(i)^q \, di \right)^{\frac{1}{q}},
\]

and consumers maximize over the aggregate consumption \( C_t \) given by

\[
C_t = \left( \int_0^1 c_t(i)^q \, di \right)^{\frac{1}{q}}.
\]

The parameter \( q \in (0, 1) \) represents the degree of complementarity for inputs in production and goods for consumption. Final goods producers maximize profits \( P_t Y_t - \int_0^1 p_t(i) y_t(i) \, di \) where \( P_t \) is the final goods price. Optimizing (see Chari, Kehoe and McGrattan (1996) or Walsh (2003)) yields the following condition on the demand for intermediate goods.

\[
y_t^d(i) = Y_t \left( \frac{P_t}{p_t(i)} \right)^{\frac{1}{1 - q}}
\]

(2)

Final good producers are competitive and make zero profits, which determines the following condition on prices.

\[
P_t = \left( \int_0^1 \frac{q}{p_t(i)^{1 - q} \, di} \right)^{\frac{1}{1 - q}}
\]
For the succeeding analysis, an important point is that a change in a single intermediate good price does not affect the final good price, since there is a continuum of goods.

### 2.2 Household decision

The consumers face a standard labor-leisure problem and maximize over consumption $C_t$, labor supplied $L_t$, and savings (deposits) $D_{t+1}$. Utility $u(C_t)$ depends on consumption and the disutility of labor is given by $w(L_t)$. Consumers are owners of the intermediate goods firms and get a share of the profits $\Pi(i)$ across all firms, though the effects on profits do not enter their optimization decision. The other relevant variables in their decision are the interest rate $r_t$ and the nominal wage $W_t$.

\[
\max_{C_t, L_t, D_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t (u(C_t) - w(L_t)) \quad \text{subject to} \quad \int_0^1 p_t(i) c_t(i) di + D_{t+1} \leq (1 + r_t) D_t + W_t L_t + \int_0^1 \Pi_t(i) di
\]

For deriving a Phillips curve style relation, the first order conditions with respect to consumption and labor of the household’s problem (3) are important. Let the Lagrange multiplier at time $t$ be $\lambda_t$, so these conditions are as follows.

\[
\lambda_t = \frac{u'(C_t)}{p_t(i)} \cdot \frac{C_t}{c_t(i)}
\]

\[
w'(L_t) = \lambda_t W_t
\]

### 2.3 Working capital requirement

The primary deviation of the present model from standard New Keynesian approaches is to require that to produce intermediate goods firms must have working capital in the form of final goods to be repaid in the next period as in Boissay (2001). Working capital is distinct from the intermediate goods and fixed capital $k_t$. Production of intermediate goods has standard Cobb-Douglas form with productivity shock $a_t$ that has mean $\bar{a} > 1$. To focus on short run dynamics\(^5\), assume $k_t = 1$ so intermediate goods production is given by $y_t(i) = a_t L_t^a$, assuming the firm receives the necessary working capital.

Intermediate goods producing firms are heterogeneous in their need for working capital $\phi_t$. Let the variable $\nu_t$ be distributed on $[0, 1]$ and represent the fraction of the size of the firm required as working capital.

---

\(^5\)Ignoring capital is common in the monetary policy literature. Woodford (Chapters 1-4, 2002) and McCallum and Nelson (2004) discuss the issue in detail.
capital to produce. Firms request funds relative to their size and independent of the productivity parameter \( a_t \) since higher productivity has an ambiguous effect on the need for financing. Hence, required working capital depends on \( \nu_t \) and \( L_t \) such that \( \phi(\nu_t) = \nu_t L_t^{\alpha} \). Firms that get financing repay \( (1 + r_t) \phi(\nu_t) \), but they cannot commit to doing so and must collateralize their debt through fixed capital and cash flows. Let \( \gamma \) be the the fraction of fixed capital and \( \mu \) the fraction of the production \( y_t(i) \) the bank can claim in case of default. Therefore, firms get financing if \( \gamma + \mu y_t(i) \geq (1 + r_t) \phi(\nu_t) \). Note that the variables in the condition are all for time \( t \) so it may be expressed in real terms. Since fixed capital does not play an important role in the present model, let \( \gamma = 0 \). Projects get financing if the fraction of production requiring financing is less than the boundary \( \bar{\nu}_t \), so that \( \nu < \bar{\nu}_t \). Substituting for \( y_t(i) \) and \( \phi(\nu_t) \) in the collateral constraint shows the boundary can be expressed as follows.

\[
\bar{\nu}_t = \min \left\{ \frac{\mu a_t}{1 + r_t}, 1 \right\} \quad (6)
\]

The productivity shock \( a_t \) appears since it affects the cash flow a firm can generate with funding but not the request for funds. A more productive firm could have a greater need for working capital to increase production or a lower need if it can substitute other inputs.

Firms produce intermediate goods only if they get financing so the need for working capital restricts production if \( \bar{\nu}_t < 1 \). Let the variable \( \nu \in [0,1] \) have distribution \( F(\nu) \) with expectation denoted \( \nu^e \) with the usual relation between expectation and distribution. Intermediate goods producing firms maximize

\[
\Pi(i) = p_t(i) \left[ F(\bar{\nu}_t) a_t - \frac{P_t}{p_t(i)} \bar{\nu}_t^x (1 + r_t) \right] L_t^{\alpha} - W_t L_t. \quad (7)
\]

This optimization is static and \( \bar{\nu}_t^x \) is realized in time \( t \) so the notation for the expectation on this term is distinct from expectations in the household’s problem (3). This profit function follows Boissay (2001) with the additional features that firm sales revenue depends on the individual good price \( p_t(i) \) and the repayment of working capital depends on the aggregate price \( P_t \). The bound determining which projects get financing enters twice since \( y_t = F(\bar{\nu}_t) a_t L_t^{\alpha} \) is the expected level of production before \( \nu_t \) is realized and \( \bar{\nu}_t^x (1 + r_t) L_t^{\alpha} \) is the expected cost of financing. Note that these terms also represent the aggregate levels of production and cost of financing across intermediate goods producing firms. Let the productivity shock be such that \( a_t > 1 + r_t \), which ensures positive production since \( F(\bar{\nu}_t) \geq \bar{\nu}_t^x \).

While the time \( t \) choice variables for the household \( \{c_t, L_t, D_{t+1}\} \) and the firm \( \{p_t(i), L_t\} \) are made before the individual firms’ needs for financing \( \nu_t \) is determined, agents know the aggregate quantities \( \bar{\nu}_t^x \) and \( F(\bar{\nu}_t) \) related to the bound on financing. Hence, the bound \( \bar{\nu}_t \) is determined simultaneous to the other
By assumption, firms pay wages whether or not they get financing. One justification is that firms have multiple projects and $\nu$ represents the fraction of projects that are financed so workers can be reassigned within the firm to financed projects. Alternatively, we could assume that producing firms pay into an unemployment insurance fund that pays workers at firms that did not get financing.

The profit function (7) shows both price and quantity rationing. If all firms get financing, $\bar{\nu}_t = 1$, the interest rate still enters the profit function, representing price rationing or the cost of financing analogous to Ravenna and Walsh (2006). If $\bar{\nu}_t < 1$ and some firms do not get financing there is quantity rationing, which directly restricts production. Furthermore, equation (6) shows that the fraction $\bar{\nu}_t$ depends on the interest rate, so there is a second credit channel for monetary policy.

3 Profit Maximization and Aggregate Supply Relations

The profit maximizing pricing decision by intermediate goods producers determines the reduced form aggregate supply relation. To show the role of the working capital requirement on inflation dynamics, it is necessary to specify the nature of the price frictions and use results from the labor and deposit markets.

Using the demand for intermediate goods (2) and the production function (7) to substitute for labor, the expected profit for intermediate goods producers (7) can be written in terms of the firm’s price, the aggregate price and output. Following Boissay (2001), let $\nu_t$ be uniformly distributed on $[0,1]$ so $F(\bar{\nu}_t) = \nu_e t = \bar{\nu}_t$.

$$
E_t(\Pi_t(i)) = E_t \left\{ p_t(i) \bar{\nu}_t \left[ a_t - \frac{P_t}{p_t(i)} (1 + r_t) \right] Y_t \left( \frac{P_t}{p_t(i)} \right)^{1-q} \frac{1}{\gamma} \left( Y_t \frac{1}{\bar{\nu}_t a_t} \right)^{\frac{1}{\alpha}} \left( \frac{P_t}{p_t(i)} \right)^{\frac{1}{\alpha} (1-q)} \right\}.
$$

The expectation is necessary since the wage depends on time $t+1$ variables as is detailed below. Therefore, the first order condition $E_t(\Pi'(p_t(i))) = 0$ with respect to the firm’s price $p_t(i)$ is

$$
E_t \left\{ q \bar{\nu}_t \left[ a_t - \frac{P_t}{p_t(i)} (1 + r_t) \right] Y_t \left( \frac{P_t}{p_t(i)} \right)^{1-q} \frac{1}{\gamma} \left( Y_t \frac{1}{\bar{\nu}_t a_t} \right)^{\frac{1}{\alpha}} \left( \frac{P_t}{p_t(i)} \right)^{\frac{1}{\alpha} (1-q)} \right\} = 0 \quad (8)
$$

Following Woodford (2000), labor is not specialized, and firms are sufficiently small to be wage takers, so changes in individual intermediate goods prices do not affect the wage directly.
### 3.1 Price setting

It is now possible to discuss the nature of the price stickiness using a preliminary linearization of the profit maximizing condition (8) above. To focus on the effects of the introduction of working capital, we eschew the more complicated forms of nominal frictions such as staggered wages (Taylor 1979, 1980) and staggered price setting (Calvo 1983) and simply assume that a fraction \(1 - \tau\) of firms must set prices a period in advance. In the micro-founded approach of Woodford (2002, section 3.1.3), such an assumption produces a New Classical Phillips curve such as (1) where inflation depends on the expectation from the previous period and the output gap. Of course, the model in this paper has extra structure, and our goal is to examine the resulting changes.

Let \(p_{1t}\) be the price for firms who can set it contemporaneously and so must satisfy \(E_t (\Pi' (p_{1t})) = 0,\) and \(p_{2t}\) be the price set a period in advance so \(E_{t-1} (\Pi' (p_{2t})) = 0.\) Linearization of both of these profit maximizing condition shows that \(\hat{p}_{2t} = E_{t-1} (\hat{p}_{1t})\), where \(\hat{p}_{1t}\) and \(\hat{p}_{2t}\) are percentage deviations from one, the normalized steady state value of prices. The aggregate price level \(\hat{P}_t\) as a percentage deviation is an average weighted according the fraction of firms that must set prices in advance so \(\hat{P}_t = \tau \hat{p}_{1t} + (1 - \tau) \hat{p}_{2t},\) and inflation \(\pi_t\) can be expressed as \(\pi_t = \hat{P}_t - \hat{P}_{t-1}.\) Manipulation of these relations (see Appendix) gives the following relation between firm level and aggregate prices and inflation.

\[
\pi_t - E_{t-1}\pi_t = \frac{\tau}{1 - \tau} (\hat{p}_{1t} - \hat{P}_t)
\]

### 3.2 Deposit market

To find a reduced form version of the profit maximizing condition, i.e. derive a Phillips curve, the wage \(W_t\) can be expressed in terms of aggregate prices and output through consideration of the markets for labor and deposits. Funds for working capital financing come from household deposits \(D_t\). Deposits are a nominal variable so the market clearing aggregate quantity of loans is given by \(D_t = P_t \phi (\nu^e_t)\). Hence, deposits can be written as follows\(^6\).

\[
D_t = P_t \nu_t L_t^\alpha
\]

As in Woodford (2002, section 3.1) the firm production decision has a unique solution. Since firms face an identical environment, in equilibrium\(^7\) firms produce the same quantities of the differentiated goods so \(y_t (i) = Y_t\) and \(c_t (i) = C_t.\) The discussion of the profit function (7) with the assumption about the uniform

---

\(^6\) Again, assume that the fraction \(\nu_t\) is uniformly distributed on \([0, 1]\).

\(^7\) The index of goods \(i\) and the fraction of rationed firms \(\nu_t\) are unrelated. By assumption, as long as a positive fraction of firms produces, i.e. \(\nu_t > 0\), then all goods are produced.
distribution of $\nu_t$ implies that aggregate output is given by

$$Y_t = \nu_t a_t L_t^\alpha.$$  \hfill (10)

Using the above expression for deposits and output and the profit for intermediate goods producing firms (7), the household budget constraint (3) in a symmetric equilibrium is

$$Y_t = C_t + \frac{D_{t+1}}{P_t}.$$  

Combining the three equations above yields the following expression for the aggregate level of consumption.

$$C_t = Y_t \left(1 - \frac{P_{t+1}Y_{t+1}}{a_{t+1}P_t Y_t}\right)$$  \hfill (11)

The fraction of output available for consumption increases with productivity but falls with changes in future nominal output due to the increased savings required for financing. The above expression indicates that the steady state fraction of output consumed is $1 - \frac{1}{\bar{a}}$. In the present setting, this fraction represents the marginal propensity to consume so a sensible value for $\bar{a}$ is well above one.

### 3.3 Labor market

Furthermore, in a symmetric equilibrium, the first order conditions (4, 5) from the household optimization problem yield the following condition on the real wage.

$$\frac{w'(L_t)}{w'(C_t)} = \frac{W_t}{P_t}.$$  \hfill (12)

To make these conditions operational, assume the following functional forms\(^8\) for the utility of consumption and the disutility of labor, $u(C_t) = \ln C_t$ and $w(L_t) = \frac{\chi}{1 + \eta} L_t^{1+\eta}$. Then the above expressions for aggregate consumption (11) and the equilibrium condition in the labor (12) determine the aggregate nominal wage.

$$W_t = \chi P_t L_t^\eta Y_t \left(1 - \frac{P_{t+1}Y_{t+1}}{a_{t+1}P_t Y_t}\right)$$  \hfill (13)

Finally, using this expression for the wage and the production function (10) to substitute for $L_t$, the profit maximizing condition\(^9\) for intermediate goods producers can be expressed in terms of prices, output, the

---

\(^8\) Both specifications are common in the literature, see Boissay (2000) and Ravenna and Walsh (2002). Since our primary goal is to linearize around a steady state, these choices are not critical.

\(^9\) The fact that we assumed a symmetric equilibrium to find $W_t$ but did not do so for the profit maximizing condition is not problematic, since the wage does not vary with individual firm prices $p_t(i)$.  

interest rate and productivity growth.

\[ E_t \left\{ q \tilde{v}_t \left[ a_t - \frac{P_t}{p_t(i)} (1 + r_t) \right] - \frac{X}{\alpha} Y_t \left( 1 - \frac{P_{t+1} Y_{t+1}}{a_{t+1} P_{t+1} Y_{t+1}} \right) \left( \frac{Y_t}{p_t a_t} \right)^{1 + \eta - \alpha} \left( \frac{P_t}{p_t(i)} \right)^{\frac{1 - \alpha}{\alpha}} \right\} = 0 \]  

(14)

### 3.4 The Linearized aggregate supply relation

The linear approximation of the above condition (14) in terms of percentage deviation from the steady state follows where \( \Delta \hat{Y}_t \) is the percent change nominal output, \( \hat{Y}_t \) is the percent deviation of output from its steady state and the others variables are similarly defined.

\[ c_y \hat{v}_t + c_a \hat{a}_t - c_r \hat{r}_t + c_y E_t \left( \Delta \hat{Y}_{t+1} \right) - c_y \hat{Y}_t - c_p \left( \hat{P}_t - \hat{p}_{tt} \right) = 0 \]  

(15)

For reasonable parameter values all the \( c \) coefficients in the above linearization are positive, as shown in the Appendix. Using the inflation adjustment equation (9), the linearization takes the form of a Phillips curve style inflation adjustment equation.

\[ \pi_t = E_{t-1} \pi_t + c_p^{-1} \left( \frac{\tau}{1 - \tau} \right) \left[ c_y \hat{Y}_t + c_r \hat{r}_t - c_y \hat{v}_t - c_y E_t \left( \Delta \hat{Y}_{t+1} \right) - c_a \hat{a}_t \right] \]  

(16)

One result from this linearization is that the interest rate enters the supply relation in a manner analogous to Ravenna and Walsh (2006), who show the existence of a cost channel arising from a cash-in-advance constraint. However, the role of the financing bound \( \hat{v}_t \) introduces a role for credit market flows of funds in the Phillips Curve. If some firms do not get financing, i.e. \( \hat{v}_t < 1 \), then the specification (6) shows that \( \hat{v}_t \) depends on both the interest rate and the productivity shock. Linearizing (6) determines the following, where \( \tilde{r} \) is the steady state interest rate$^{10}$.

\[ \hat{v}_t \approx \frac{\mu a}{1 + \tilde{r}} \left( \hat{a}_t - \frac{1}{1 + \tilde{r}} \hat{r}_t \right) \]

The interest rate thereby affects the supply decision in two ways, and the inclusion of \( \hat{v}_t \) in the aggregate supply relation represents a second credit channel for monetary policy. A higher \( r_t \) increases the cost of working capital and reduces the possibility of getting financing in the first place.

The inclusion of the fraction of firms receiving financing in the Phillips Curve has important empirical implications. First, a measure of \( \nu_t \) should be constructed from data on credit market conditions. The Phillips Curve (16) could be estimated without such a measure, appealing to the dependence of \( \hat{v}_t \) on \( \hat{r}_t $^{10}$The steady state interest rate is determined by the IS relation for this model. See Appendix for details.
shown above, but this implies a relatively larger coefficient on $r_t$ than for the specification of Ravenna and Walsh (2006). Furthermore, the restriction that $\nu_t \leq 1$ suggests a regime switching approach to estimation. If all firms get financing, $\nu_t = 1$ in some periods, but the collateral constraint binds representing a "flight to quality" in other periods, the variable $\nu_t$ should be modeled using a Markov switching approach as in Hamilton (1989).

An unusual feature of the supply relation (16) is the presence of expected changes in nominal output $E_t(\Delta \hat{Y}_{t+1})$ with a negative coefficient, due to expectations about future deposit market conditions. If nominal output is expected to increase, the current demand for deposits rises according restricting consumption and current prices. Superficially, there is a similarity with the New Keynesian Phillips curve that depends on expected inflation, as expected nominal output could be decomposed into inflation and output growth. However, in the present model expected future inflation enters with a negative coefficient that is much less than one\(^\text{11}\) in magnitude in contrast to the New Keynesian specification.

The productivity shock $\hat{a}_t$ enters the supply relation (16) through multiple channels, since it affects production, the demand for labor, the expected need for financing and the fraction of firms that get financing. For monetary policy considerations, supply shocks are critically important since they give rise to the inflation-output trade-off in the policymaker’s decision. The Phillips curve relation in Ravenna and Walsh (2006) does not have an explicit stochastic term as it is expressed in terms of the deviation of output and interest rate from their flexible price values, which depend on the shock to productivity\(^\text{12}\). They point out that, even without a productivity shock, the presence of the interest rate in the Phillips Curve and a demand shock in the IS relation means that the policymaker faces the trade-off analogous to the one arising from a supply shock in the standard New Keynesian model.

The supply relation (16) does show policy effectiveness even though there are minimal frictions in the pricing process. A policymaker with influence over interest rates has a direct impact on firm supply decisions.

\section{Conclusion}

This paper presents a New Keynesian general equilibrium model with a credit constraint on firms more sophisticated than the one found in Ravenna and Walsh (2006). The presence of quantity rationing, due to the collateral requirements, opens a second credit channel. The fraction of firms receiving financing enters the linearized aggregate supply relation. Changes in nominal output also enter in a counter-intuitive way.

\(^\text{11}\)The end of section 3.2 gives an argument for a large $\hat{a}$ which implies a small magnitude of the coefficient $c_Y$ as shown in the Appendix.

\(^\text{12}\)The Phillips Curve relation in the present work could be given in terms of deviation from flexible price values, though it would require the construction of a the flexible price fraction $\nu_f$ of firms receiving financing.
There are a number of avenues for further work in this area. The presence of the faction of firms receiving financing has important empirical implications for the estimation of the Phillips Curve (16). How the fraction \( \nu_t \) could be interpreted using data on flows of funds in the credit market is an important issue. Exploring the implications for optimal monetary policy is another goal. More broadly, this model has the potential to study the role of credit markets on the macro-economy and the effects of policy in light of financial market considerations.

This paper is part of a larger research project with the primary goal of studying a model with multiple steady states arising from financial factors. In this model, if the fraction \( \mu \) of cash flow that lenders can claim in the case of default, depends endogenously on the size of the firm, there could be an extra steady state with a higher degree of quantity rationing and lower output. Such a model could describe recessions driven by financial factors.

Appendix

From the profit maximizing condition (8), the linearizations of \( E_t (\Pi' (p_{1t})) = 0 \) and \( E_{t-1} (\Pi' (p_{2t})) = 0 \) take the form, respectively

\[
\tilde{c}_p \hat{p}_{1t} + \tilde{c}_w \hat{W}_t + \tilde{c}_y \hat{Y}_t + \tilde{c}_r \hat{R}_t + \tilde{c}_a \hat{a}_t = 0,
\]

\[
\tilde{c}_p \hat{p}_{2t} + E_{t-1} \left[ \tilde{c}_w \hat{W}_t + \tilde{c}_p \hat{P}_t + \tilde{c}_y \hat{Y}_t + \tilde{c}_r \hat{R}_t + \tilde{c}_a \hat{a}_t \right] = 0.
\]

Taking the expectation \( E_{t-1} \) of the first equation and subtracting the second implies that \( E_{t-1} \hat{p}_{1t} = \hat{p}_{2t} \).

The aggregate price level \( \hat{P}_t \) is an average weighted according to the fraction \( 1 - \tau \) of firms that must set prices one period in advance so

\[
\hat{P}_t = \tau \hat{p}_{1t} + (1 - \tau) \hat{P}_{2t} = \tau \hat{p}_{1t} + (1 - \tau) E_{t-1} \hat{p}_{1t}.
\]

Since inflation \( \pi_t \) is simply \( \pi_t = \hat{P}_t - \hat{P}_{t-1} \), the forecast error for inflation can be written as \( \pi_t - E_{t-1} \pi_t = \hat{P}_t - E_{t-1} \hat{P}_t \). Substituting using the relationship between \( \hat{P}_t \) and \( \hat{p}_{1t} \) twice yields

\[
\pi_t - E_{t-1} \pi_t = \tau (\hat{p}_{1t} - E_{t-1} \hat{p}_{1t})
\]

\[
= \tau \left( \hat{p}_{1t} + \frac{1}{1 - \tau} \left( \tau \hat{P}_{1t} - \hat{P}_t \right) \right),
\]

which is equivalent to the inflation adjustment equation (9).

To determine the steady state values in terms of the parameters, it is necessary to find the IS style relation
using the first order conditions of the household maximization problem (3) with respect to consumption (4) and deposits $D_{t+1}$

$$\lambda_t = \beta E_t [\lambda_{t+1} (1 + r_{t+1})].$$

In equilibrium when $c_t (i) = C_t$ and $p_t (i) = P_t$, these two first order condition give the following demand relation substituting to eliminate the Lagrange multiplier $\lambda_t$.

$$1 = \beta E_t \left[ (1 + r_{t+1}) \frac{P_t C_t}{P_{t+1} C_{t+1}} \right]$$

It is possible to use the consumption relation (11) to find a relation between output and the real interest rate, but the above is sufficient to determine the steady state interest rate

$$\tilde{r} = \frac{1 - \beta}{\beta}.$$

Assuming there is some quantity rationing at the steady state, the fraction of firms given by (6) shows that

$$\tilde{\nu} = \frac{\mu \tilde{a}}{1 + \tilde{r}}.$$

Next, the equilibrium first order condition of the profit function (7) with respect to labor and the wage condition (13) can be used to substitute for $W_t$ to give the following condition on the steady state values.

$$\chi \tilde{L}^\alpha \tilde{C} = \tilde{\nu} (\tilde{a} - 1 - \tilde{r}) \tilde{L}^{\alpha - 1}$$

The steady state value of output is determined by the ex-post production equation (10) so $\tilde{Y} = \tilde{\nu} \tilde{a} \tilde{L}^\alpha$.

The relationship between steady state consumption and output is seen in equation (11), which determines $\tilde{C} = \tilde{Y} \left(1 - \frac{1}{\tilde{a}}\right)$. Using this expression with the condition above allows us to solve for $\tilde{Y}$.

$$\tilde{Y} = \left[ \chi^{-1} \left(1 - \frac{\tilde{r}}{\tilde{a} - 1}\right) \right]^{\frac{\alpha}{1 + \eta \tilde{r}}}$$

The steady state values $\tilde{a}, \tilde{r}, \tilde{\nu}$ and $\tilde{Y}$ are sufficient to find the values of the coefficients in the Phillips Curve (16) in terms of the parameters. The linearization (15) of the profit maximization condition (14)
around the steady steady state determines the following coefficients.

\[ c_p = \mu \tilde{a} \left[ q - \frac{\tilde{a}(1 - \alpha)}{\tilde{a}^2(1-q)} \left( \beta - \frac{1 - \beta}{\bar{a} - 1} \right) \right] \]

\[ c_Y = \frac{\chi}{\alpha} \left( 1 - \frac{1}{\tilde{a}} \right) \left[ \frac{\mu \beta \tilde{a}^2}{\chi} \left( 1 - \frac{1 - \beta}{\beta(\bar{a} - 1)} \right) + \left( \frac{1 + \eta}{\alpha} - 1 \right) \left( \chi^{-1} \left( 1 - \frac{1 - \beta}{\beta(\bar{a} - 1)} \right) \right)^{1 - \frac{\alpha}{1 + \eta}} \right] \]

\[ c_r = q\mu \tilde{a} \left( 1 - \beta \right) \]

\[ c_r = q\mu \tilde{a} \left( \beta \tilde{a} - 1 \right) + \left( \frac{1 + \eta}{\alpha} - \alpha \right) \left( \tilde{a} - 1 - \frac{1 - \beta}{\beta} \right) \]

\[ c_r = \frac{\mu}{\alpha} \left( \beta - \frac{1 - \beta}{\bar{a} - 1} \right) \]

\[ c_a = q\mu \beta \tilde{a}^2 + \frac{\mu}{\alpha} \left( \frac{1 + \eta}{\alpha} - 1 \right) \left( \tilde{a} - 1 \right) \left( \beta - \frac{1 - \beta}{\bar{a} - 1} \right) \]
References


