Persuasive Advertising with Network Effects

(preliminary, incomplete and rather terse)

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Abstract

If there’s lots of advertising, you’re paying too much. We study a linear city model where status is a consumption externality that depends on the number of other people using the product. Advertising affects utility both directly and through the magnitude of the externality. A major finding is that when a firm’s advertising positively affects the status of its own product then the optimal level of advertising under competition is decreasing with the strength of the status effect. However, if firms collude the opposite is true and advertising increases with the magnitude of the status effect.

Many economists have described the network externality on utility created by status. A consumer’s desire for a good is frequently increased if many other consumers also use that good. Some firms use advertising to affect the status of products of their own and/or their competitors.

Firms often spend large sums on advertising which has little or no informational content. Some industries such as soft drink or athletic shoe manufacturers advertise to improve consumers’ impressions about the quality of the product and to increase the status conferred by their products. This model studies such advertising and the conditions that determine levels of advertising in an economy with product differentiation and network effects. The primary result is that firms will often advertise more when they are colluding than they would in a competitive environment.

1 The Model

The model is a unit length linear city with firms X and Y symmetrically located on the left and right, respectively. Consumers are uniformly distributed on the segment and pay quadratic travel costs. Each consumer located at z buys one good from firm X or Y and receives utility,

\[ U_x(z) = F_x + \theta_x \hat{z} - p_x - dz^2 \] (1)
\[ U_y(z) = F_y + \theta_y(1 - \hat{z}) - p_x - d(1 - z)^2. \] (2)

Throughout the model we assume that \( F \) is large enough to ensure that the market is covered. The indifferent consumer is located at \( \hat{z} \), which also measures the market share of firm X or the fraction of the population buying from X. Similarly, \( 1 - \hat{z} \) is the market share for firm Y. The terms with \( \theta \) represent effect of status on the utility of a product. Utility from consuming a good rises as the fraction of consumers buying the same product rises. The parameters \( \theta_x \) and \( \theta_y \) measure the strength of the status effect for the products of each firm. The first term \( F \) represents the consumer’s perception of the value of the good. The prices are \( p_x \) and \( p_y \), and the terms with \( d \) measure the travel costs, as usual.

Advertising enters the utility functions two ways. It directly changes the perceived value of a product through \( F \) and indirectly alters status through \( \theta \). At some level of advertising \( a \), the cost to the firm is

\[ C(a) = \frac{a^2}{2}. \] (3)

The direct effects of advertising \( a_x \) for firm X and \( a_y \) for firm Y are

\[ F_x(a_x) = \gamma a_x + f \] (4)

\[ F_y(a_y) = \gamma a_y + f \] (5)

where \( \gamma \) captures the strength of this effect. The impact of advertising on status is

\[ \theta_x(a_x, a_y) = \alpha a_x - \beta a_y \] (6)

\[ \theta_y(a_x, a_y) = \alpha a_y - \beta a_x. \] (7)

A firm’s advertising can positively affect the status of its own product and negatively affect the status of its opponent’s product. These positive and negative effects are measured by the parameters \( \alpha \) and \( \beta \), respectively. All the parameters \( \alpha, \beta, \gamma \) and \( f \) are assumed to be positive.

We take the marginal cost of production to be zero throughout. Profits for the firms are

\[ \pi_x(a_x, a_y, p_x, p_y) = p_x \hat{z} - C(a_x) \] (8)

\[ \pi_y(a_x, a_y, p_x, p_y) = p_y(1 - \hat{z}) - C(a_y). \] (9)

These are simply the price a firm charges multiplied by its market share minus the costs of its advertising.

The location of the indifferent consumer \( \hat{z} \) is determined by \( U_x(\hat{z}) = U_y(\hat{z}) \), which gives us

\[ \hat{z} = \frac{F_x - F_y + p_y - p_x + d - \theta_y}{2d - (\theta_x + \theta_y)}. \] (10)
If both firms set identical prices and advertising levels this expression will reduce to 
\( \hat{z} = \frac{1}{2} \), but firms may increase their market share above one half if they undercut their opponents price or advertise at higher level. However, in the game, an aggressive advertising strategy by one firm may induce the other firm to counter with a low price.

2 Competition

Under competition, the firms play a two stage game. First, each firm simultaneously chooses its level of advertising \( a_x \) or \( a_y \), then, after observing the choice of a’s, they choose prices \( p_x \) and \( p_y \). We will solve by backwards induction, maximizing over prices, given fixed levels of advertising, then maximizing over advertising levels using equilibrium prices. Given \( a_x \) and \( a_y \) fixed, each firm maximizes profits over its own price. These first-order conditions are

\[
\hat{z} + p_x (\theta_x + \theta_y - 2d)^{-1} = 0 \tag{11}
\]

\[
1 - \hat{z} + p_y (\theta_x + \theta_y - 2d)^{-1} = 0, \tag{12}
\]

which can be solved for the competitive prices

\[
p_x^* = d + \frac{1}{3} (F_x - F_y - \theta_x - 2\theta_y) \tag{13}
\]

\[
p_y^* = d + \frac{1}{3} (F_y - F_x - \theta_y - 2\theta_x). \tag{14}
\]

As in the textbook linear city model, the lower the travel cost \( d \), the closer the economy gets to Bertrand competition, which implies lower prices and profits. Similarly, as in Grillo, Shy and Thisse [1999], the greater the affect of status, the fiercer the competition, since both prices are falling as the \( \theta \)’s rise. Higher status compensates for the travel costs and increase the price elasticity of demand. However, positive levels of advertising are possible in equilibrium. If the perceived value of the products \( F_x \) and \( F_y \) are different, the firm with higher value can charge a higher price. Both firms will have incentive to advertise more than its competitor to raise the value of its product. Further, advertising imposes a negative externality on the status of the opponents product, for example, the advertising of firm X will lower \( \theta_y \). Again, if one firm advertises more than the other, it may be able to charge a higher price. So firms might advertise under competition even though higher advertising will lead to lower prices and profits in equilibrium.

From these equilibrium prices, we can write the market shares as

\[
\hat{z} = \frac{p_x^*}{2d - (\theta_x + \theta_y)} \tag{15}
\]
\[ 1 - \hat{z} = \frac{p_y^*}{2d - (\theta_x + \theta_y)}, \]  

and so the profits may be written

\[ \pi_x(a_x, a_y) = p_x^2 (2d - (\theta_x + \theta_y))^{-1} - \frac{a_x^2}{2}, \]  

\[ \pi_y(a_x, a_y) = p_y^2 (2d - (\theta_x + \theta_y))^{-1} - \frac{a_y^2}{2}. \]  

To find equilibrium advertising levels, we will maximize each firms’ profits over its own \( a \), holding its opponents’ level fixed to find best-reply mappings. The first order condition with respect to \( a_x \), for firm X is

\[ \frac{2}{3} (\gamma - \alpha + 2\beta) \hat{z} + (\alpha - \beta) \hat{z}^2 = a_x. \]  

The first term in this equation accounts for the change in price due to a change in advertising, while the second shows the effect on market share from the same change. In equilibrium, a change in the effectiveness of advertising on status \( \alpha \) or \( \beta \) will have opposite impacts on price and market share. For example, if the effectiveness of advertising on the status of a firm’s own good good \( \alpha \) rises, the firms will want to advertise more to achieve greater market share, but will also have a competing incentive to reduce advertising to keep the price from falling. A change in \( \beta \), however, would have the exact opposite effect on the response of price and market share to changes in advertising levels.

We ignore the possibility that one firm will try to drive the other out of business and concentrate on the symmetric equilibrium. The symmetric equilibrium will be such that \( a_x = a_y \) and \( \hat{z} = \frac{1}{2} \). The advertising level for both firms at this competitive, symmetric Nash equilibrium is therefore,

\[ a_N = \frac{1}{12} (4\gamma - \alpha + 5\beta). \]  

The Nash level of advertising falls with the effectiveness of a firm’s advertising on its products status \( \alpha \) as higher status makes the market more competitive, lowering prices and profits. Assuming advertising levels cannot be negative, if \( 4\gamma - \alpha + 5\beta < 0 \), the equilibrium \( a_N = 0 \).

In fact, this will occur in the special case when \( \gamma, \beta = 0 \) and advertising impacts only the advertising firm’s own status through \( \alpha \). Here, with neither firm advertising, there is still no profitable deviation to a positive level of advertising. If \( a_y = 0 \) in this case, the equilibrium prices are

\[ p_x^*(a_x) = d - \frac{\alpha}{3} a_x \]  

\[ p_y^*(a_x) = d - \frac{2\alpha}{3} a_x. \]
If firm X deviates to a positive \( a_x \) then firm Y undercut X’s price in the next stage to preserve some of his market share. Examining firm X’s profit with these restrictions, 
\[
\frac{\partial \pi_x}{\partial a_x} (a_x = 0) = -\frac{\alpha}{12} < 0
\]
so it should minimize its advertising. While the advertising firm’s market share will increase, in the second stage the non-advertising firm will undercut the price of the advertising firm, making deviation suboptimal.

For competitive firms to advertise in equilibrium the effects on the value of the product \( \gamma \) and on the opponent’s status \( \beta \) must be sufficiently strong. Also, to ensure that the market is covered the following condition must hold:
\[
F \geq \frac{3}{2} (d - \theta) \quad \text{where } \theta = (\alpha - \beta) a_N \tag{23}
\]
This condition can always be satisfied by setting \( f \), the product value in the absence of advertising, high enough.

Summarizing, we have

**Proposition 1** If firms compete and the market is covered, the symmetric Nash equilibrium is characterized by

i) \( a_N = \frac{1}{12} (4\gamma - \alpha + 5\beta) \)

ii) \( p = d + (\alpha - \beta) a_N \)

iii) \( \tilde{z} = \frac{1}{2} \)

The market is covered if \( F \geq \frac{3}{2} [d - (\alpha - \beta) a_N] \).

### 3  Collusion

If firms collude, they split the market and set prices and advertising levels to maximize total profits. The price will be set so that the indifferent consumer at \( \tilde{z} = \frac{1}{2} \) will pay his reservation price, so \( U_x (\tilde{z}) = 0 \) which implies
\[
p = F - \frac{1}{4} (d - 2\theta), \tag{24}
\]
and total profit is
\[
\pi (a) = p - a^2. \tag{25}
\]
Maximizing total profit over \( a \) yields the collusive level of advertising over the whole market. Each firm will advertise at level \( a/2 \).

**Proposition 2** The profit maximizing collusive agreement on prices and advertising levels is characterized by

i) \( a_C = \frac{1}{4} (2\gamma + \alpha - \beta) \)

ii) \( p_C = 2a_C^2 - \frac{d}{4} \)

iii) \( \tilde{z} = \frac{1}{2} \).
Here, advertising is increasing in $\alpha$, since, in an non-competitive setting, additional status allows firms to charge a higher price and achieve higher profits. Both firms agree to symmetric prices and advertising levels so market share is not an issue.

The parameters describing the effect of advertising play different roles in the collusive and competitive situations. Not surprisingly, if advertising is more effective in raising the perceived quality of a product, a higher $\gamma$, both competitive and collusive levels of advertising will increase, though the increase will be greater under collusion as competitive forces dampen the impact. The positive effect on status $\alpha$ and the negative effect on the status of the opponent’s status $\beta$ have opposite effects in the two cases. The competitive advertising level falls with $\alpha$ as competition becomes more intense, but a rise in $\beta$ would have a strong positive impact on the Nash level of advertising. Such an increase in $\beta$ would lower the opponents status and reduce the overall ferocity of competition, leading to higher prices and profits. However, under collusion, the firms seek to maximize the utility to the consumer so as to charge a higher price. Since they will coordinate and agree to advertise at the same level, firms seek to increase their opponent’s status as well as their own. Therefore the collusive level of advertising rises with $\alpha$ but falls with $\beta$.

We can also compare the different advertising levels directly.

**Proposition 3** The collusive level of advertising is greater than the total competitive level, $a_C > 2a_N$, if and only if $5\alpha > 2\gamma + 13\beta$.

If a firm’s advertising has a large impact on the status of its product (large $\alpha$) relative to the other parameters for advertising, collusive advertising levels will be higher than competitive levels. Alternatively, in an economy with large positive status effects of advertising, a high level of advertising is evidence of collusion. Such a situation corresponds to the athletic shoe industry where Nike and Reebok spend huge sums on advertising directed toward increasing the status associated with their products.

A minority of advertising directly attacks the goods of a competitor so taking $\alpha$ to be significantly larger than $\beta$ seems to be a reasonable assumption. The situation where negative advertising plays a big role (high $\beta$) is also interesting, though probably less common in practice. If advertising has a large negative impact on an opponent’s products compared with the other parameters, competitive level of advertising will be higher than collusive. This case may correspond to political advertising where expenditures are large, negative advertising is effective and candidates are competing strongly for votes.

## 4 Conclusion

The model in this paper shows a simple way to introduce persuasive advertising into a spatial duopoly with network effects. The discussion has centered on status as the
particular consumption externality in the model and advertising effect on status, but
the model may also be interpreted with other network effects such as compatibility
of technologies.

Advertising has three roles, the effect of the perceived value of a product, the
positive effect on the advertiser’s status and the negative effect on the opponent’s
status. The strengths of these effects are measured by $\gamma, \alpha$ and $\beta$, respectively. We
show equilibrium levels of advertising in both competitive and collusive environments
and interpret the associated comparative statics and comparisons of advertising levels.
The major findings are:

- A stronger direct effect of advertising on the value of a product increases equi-
librium advertising levels in both the competitive and collusive settings.

- Under competition, the equilibrium level of advertising falls as the strength
of the positive effect on status rises, while equilibrium advertising rises with
strength of the negative effect on the opponent’s status.

- Under collusion, higher status increases the price the firms can charge so equi-
librium advertising rises with the strength of the positive effect on status, while
falling with the strength of the negative effect.

- If the positive effect on status of advertising is sufficiently strong relative to
the other effects, equilibrium advertising under collusion will be greater than
under competition. Conversely, if the negative effect dominates, competitive
advertising levels will be higher.

A possible use of this model would be to detect price collusion by examining the
level of advertising. In a market with large positive affects on status, high levels of
advertising would be evidence of price collusion. In a different context, expenditures
on advertising in political campaigns give an indication of the role to status effects
and negative advertising in particular. The optimal expenditure on advertising to
effect the status of goods depends crucially on the environment.